Berowra PS Round: Are we developing students' conceptual understandings and skills in number?

The reading for the Berowra round focuses on two fundamental aspects of the problem of practice. The first section focuses on conceptual understanding and features the ground-breaking work of Di Seamon, which has been adopted by the Victorian and South Australian school systems and is being trialled in many schools in NSW.

The second section focuses on the qualities of mathematical tasks and is authored by eminent mathematical educator, Emeritus Professor Peter Sullivan. In our rounds work we inevitably find that it is the qualities of the task that determine student learning.

Working with the Big Ideas in Number - Dianne Siemon, John Bleckl, Denise Neal (extracts)

One of the main aims of school mathematics is to create mental objects in the mind's eye of children which can be manipulated flexibly with understanding and confidence. A prolonged reliance on inefficient strategies such as "make-all-count-all" or "counting-by-ones" is both developmentally dangerous and professionally irresponsible.

"Number Sense" refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations. It results in a view of numbers as meaningful entities and the expectation mathematical manipulations and outcomes should make sense. Those who use mathematics in this way continually utilise a variety of internal "checks and balances" to judge the reasonableness of numerical outcomes.

Why Focus on Big Ideas?

Students need to learn mathematics in ways that enable them to recognise when mathematics might help to interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable in the context (National Numeracy Review Report)

It has been found that this typical textbook method is unlikely to be conducive to learning mathematics in the way suggested by the National Numeracy Review (Commonwealth of Australia, 2008). A focus on the big ideas is needed to 'thin out' the over-crowded curriculum (National Mathematics Advisory Panel, 2008) and create opportunities to rethink and transform existing approaches to the teaching and learning of mathematics.

Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise. (National Council of Teachers of Mathematics, 2000)

A focus on big ideas and the links between them is needed to strengthen student understanding and help deepen teacher knowledge and confidence for teaching mathematics. The importance of this has been demonstrated by research on the characteristics of effective teachers of mathematics. For instance, effective teachers:

- recognise the connections between different aspects and representations of mathematics;
- Ask timely and appropriate questions,
- Facilitate and maintain high-level conversations about important mathematics,
- Evaluate and respond to student thinking during instruction,
- Promote understanding, help students make connections, and
- Target teaching to ensure key ideas and strategies are understood.

'Big Idea' in School Mathematics

More recently, the Awareness of the "Big Ideas in Mathematics Classrooms Project" (Kuntze et al., 2009), which is aimed at "encouraging teachers' reflections on overarching concepts in mathematics and on their potential for learning", has identified four characteristics of big ideas. These can be summarised as ideas that have high potential for building conceptual understanding, meta-knowledge about mathematics as a science, meaningful communication strategies, and professional reflection. Examples of big ideas from include 'using multiple representations', 'giving arguments or proving' and 'dealing with infinity'. While these are undoubtedly important indicators of mathematical reasoning and best teaching practice, it is not clear how these translate to learning trajectories that could be used to inform teaching and support mathematics learning over time.

For the purposes of the *Assessment for Common Misunderstandings* (Siemon, 2006) and the *Developmental Maps* (Siemon, 2011a) which were developed for the Victorian Department of Education and Early Childhood Development, and the *Interview for Student Reasoning*, NSW Department of Education a 'big idea' in mathematics:

- is an idea, strategy, or way of thinking about some key aspect of mathematics without which, students' progress in mathematics will be seriously impacted;
- encompasses and connects many other ideas and strategies
- provides an organisational structure or frame of reference that supports further learning and generalizations
- cannot be clearly defined but can be observed in activity (Siemon, 2006, 2011).

The Role of Assessment

Scaffolding student learning is the primary task of teachers of mathematics. However, this cannot be achieved without accurate information about what each student knows already and what might be within the student's grasp with some support from the teacher and/or peers. This not only requires a clear understanding of the key ideas, representations and strategies in school mathematics, how they are connected and how they might be acquired over time, it also requires assessment techniques that expose student thinking, interpretations of what different student responses might mean, and some practical ideas to address the learning needs identified. As we have seen above, this is particularly important in relation to a relatively small number of 'big ideas' and strategies in Number.

The *Assessment for Common Misunderstanding* tools were developed for the Victorian Department of Education and Early Childhood Development to address this need. They draw on research-based tasks and represent what Callingham has referred to as productive assessment, in that they provide useful, timely, appropriate information fit for purpose. Based on earlier work with pre-service teachers and schools in the Northern Territory, the tools were developed to help teachers better "understand and monitor their individual students' developing strategies and particular learning needs" in relation to a small number of very big ideas in Number without which student's progress in mathematics will be severely restricted. These ideas are summarised in Table 1.

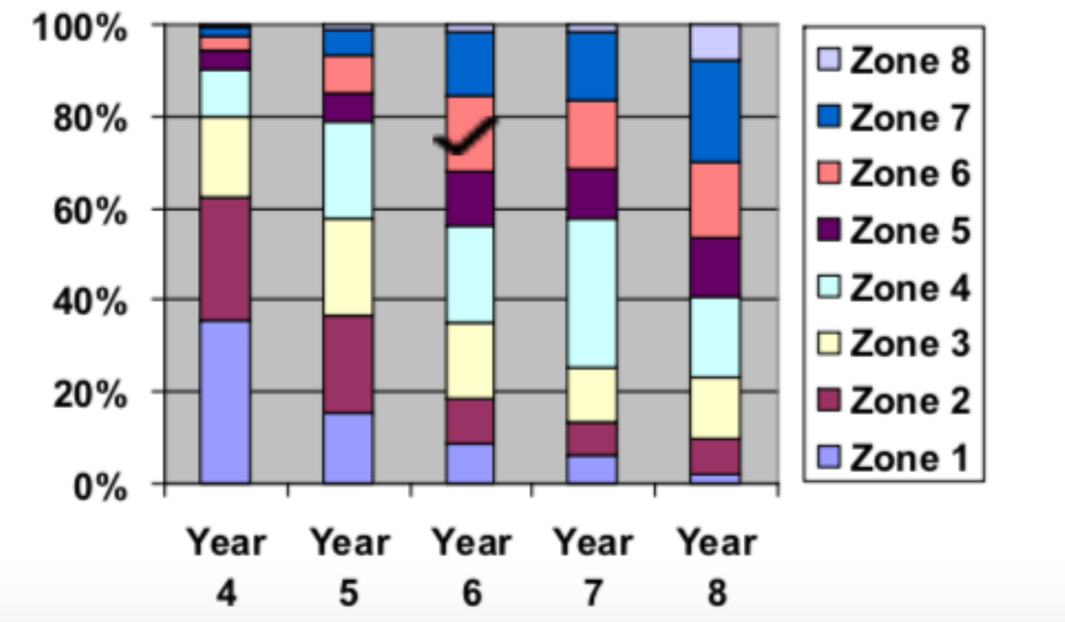
Table 1. Big ideas in Number by stages of schooling (Siemon, 2006)

By the end of:	Big Idea	Indicated by:
Foundation Year	Trusting the Count	Access to flexible mental objects for the numbers to ten based on part-part-whole knowledge derived from subitising and counting (e.g., know that 7 is 1 more than 6, 1 less than 8, 5 and 2, 2 and 5, 3 and 4 without having to make or count a collection of 7)
Year 2	Place-value	Capacity to recognise and work with place-value units and view larger numbers as counts of these units rather than collections of ones (e.g., able to count forwards and backwards in place-value units)
Year 4	Multiplicative Thinking	Capacity to work flexibly with both the number in each group and the number of groups (e.g., can view 6 eights as 5 eights and 1 more eight). Recognises and works with multiple representations of multiplication and division (e.g., arrays, regions and 'times as many' or 'for each' idea).
Year 6	(Multiplicative) Partitioning	Ability to partition quantities and representations equally using multiplicative reasoning (e.g., a fifth is smaller than a quarter, estimate 1 fifth on this basis then halve and halve remaining part again to represent fifths), recognise that partitioning distributes over previous acts of partitioning and that numbers can be divided to create new numbers
Year 8	Proportional Reasoning	Ability to recognise and work with an extended range of concepts for multiplication and division including rate, ratio, percent, and the 'for each' idea, and work with relationships between relationships
Year 10	Generalising	Capacity to recognise and represent patterns and relationships in multiple ways including symbolic expressions, devise and apply general rules

Why Big Ideas in Number?

Teachers routinely point to Number as the most difficult aspect of the school mathematics to teach and learn. This is reflected in the time spent on number in the school mathematics curriculum and evident in the data from the MYNRP, which used rich assessment tasks to explore number sense, measurement and data sense, and space sense in a structured sample of 6859 Year 5 to 9 students in 1999-2001. The results of this large-scale study found that there was as much difference in numeracy achievement within schools as between schools, that in any one year level there was there was up to an 8 year range in ability, and the needs of 'at risk' learners were not being met. A key finding of the MYNRP was that the differences in performance were almost entirely due to difficulties with larger whole numbers, decimals, fractions, multiplication and division, and proportional reasoning, collectively recognised as multiplicative thinking.

As a consequence, the *Scaffolding Numeracy in the Middle Years* [SNMY] project was designed to explore the development of multiplicative thinking in Years 4 to 8 using rich tasks and partial credit items. Rasch modeling was used to analyse the responses of just under 3200 students in three school clusters (one secondary school and three or more associated primary schools), two in Victoria and one in Tasmania. A *Learning and Assessment Framework for Multiplicative Thinking* [LAF] was identified on the basis of this analysis comprising eight hierarchical zones ranging from additive, count-all strategies (Zone 1) to the sophisticated use of proportional reasoning (Zone 8) with multiplicative thinking not evident on a consistent basis until Zone 4. The proportion of students by Zone by Year level is shown in Figure 1.



The results of the SNMY confirmed that there was an 8-year range in achievement at each year level and when the LAF Zones were analysed against curriculum expectations, it was evident that up to 40% of Year 7 and 8 students performed below curriculum expectations and at least 25% were well below expected level.

This discrepancy is unacceptable in a country that prides itself on providing opportunities for all. Multiplicative thinking is a key indicator of success in school mathematics in the middle years and as such it is imperative that the key ideas and strategies that underpin the transition from additive to multiplicative thinking are clearly articulated and understood by teachers and curriculum developers. A focus on the big ideas in number is essential to inform more targeted approaches to the teaching and learning of mathematics to ensure that all students have the opportunity to deepen their understanding and participate fully and effectively in school mathematics.

In working with the language of the Big Ideas in Number, teachers have become aware that for many students talking about mathematics strategies and learning is difficult. These students need explicit scaffolding and support to develop their mathematical vocabulary and the confidence to use it in front of their classmates.

As teachers continue their professional learning journey, there is potential for re-visiting the big ideas in number in new and exciting ways, emphasising the explicit teaching focus for teachers and the importance of tasks that are selected to focus on proficiencies. Schools have seen the potential of teacher professional learning based on in-depth knowledge of student understanding of key ideas in number. The big ideas in number provide a valuable framework for exploring questions such as "What is important?" and "What will give us the greatest leverage in improving student outcomes?" Indeed, these big ideas are influencing and engaging teachers in new learning for themselves and their students!

The Role of Mathematical Tasks

FROM "TEACHING MATHEMATICS: USING RESEARCH INFORMED STRATEGIES, PETER SULLIVAN, AUSTRALIAN COUNCIL FOR EDUCATIONAL RESEARCH, 2011

Whether in the context of developing practical or specialised mathematics, or in finding ways to encourage the breadth of mathematical actions, or in seeking to engage students in learning mathematics, the key decision that the teacher makes is the choice of task. This section outlines a rationale for the importance of appropriate tasks, illustrates some exemplary types of tasks that have been found to be useful for teachers in facilitating the learning of their students, explains some constraints teachers may experience when using challenging tasks, and describes some students' views on tasks.

Based on extensive research on the impact of mathematical tasks on student learning, a model of task identification and use was presented by Stein, Grover and Henningsen proposes that the features of the mathematical task as set up in the classroom, and the cognitive demands it makes of students, are informed by the mathematical task as represented in curriculum materials. These are, in turn, influenced by the teacher's goals, subject-matter knowledge and their knowledge of their students. This then informs the mathematical task as experienced by students which creates the potential for their learning.

The teacher determines the learning goals which they hope to have their students achieve and the types of mathematical actions in which the students will engage, noting the levels of student readiness – choosing the appropriate tasks is the next step. It is critical that teachers are mindful of the pedagogies associated with the task, and are ready to implement them. The process of converting tasks to learning opportunities is enhanced when students have opportunities to make decisions about either the strategy for solving the task or the process they will adopt for addressing the task or both. In addition, it is expected that the task will provide some degree of challenge, address important mathematical ideas and foster communication and reasoning. It is only tasks with such features that can stimulate students to create knowledge for themselves.

Why tasks are so important

Many commentators have argued that the decisions teachers make when choosing tasks are critical. Christiansen and Walther (1986) argued that the mathematical tasks that are the focus of classroom work and problem solving determine not only the level of thinking by students, but also the nature of the relationship between the teacher and the students.

In terms of the mathematical actions described by Kilpatrick et al. (2001), it is not possible to foster adaptive reasoning and strategic competence in students without providing them with tasks that are designed to foster those actions.

Drawing on an extensive program of research on student self-regulation in the United States of America, Ames (1992) argued that teachers can influence students' approach to learning through careful task design. In synthesising task characteristics suggested by other authors, she suggested the main themes were the benefits of posing a diversity of tasks types, presenting tasks that are personally relevant to students, tasks that foster metacognitive development and those that have a social component.

Carole Ames further argued that students may benefit if teachers direct attention explicitly to the longer term goals of deep understanding, linking new knowledge to previous knowledge, as well as to its general usefulness and application. She urged a focusing on the mastery of the content rather than performance to please the teacher or parents, or even students' self-esteem through any competitive advantage. Ames (explained the connection between student motivation, their self concept and their self goals, and argued that it is possible to foster positive student motivation through the provision of tasks for which students see a purpose.

Ames' findings are complemented by suggestions about tasks from Gee (2004), who formulated a set of principles for task design, derived from the analysis of computer games that had proven engaging for children and adolescents. Those of his principles that relate to mathematics task formulation were for:

- learners to take roles as 'active agents' with control over goals and strategies
- tasks to be 'pleasantly frustrating' with sufficient, but not too much challenge
- skills to be developed as strategies for doing something else rather than as goals in themselves.

While it is difficult to identify mathematics classroom tasks that incorporate all of these characteristics, both Gee's and Ames' recommendations provide a suitable standard to which teachers should aspire.

The following subsection describes some different types of mathematics tasks, including those that focus on developing procedural fluency, those that use a model or representation, and those that use authentic contexts. It also describes two types of open-ended tasks, and tasks that progressively increase the complexity of the demand on students. The discussion of the types of tasks is intended to indicate to teachers some options for the tasks they pose, and also the range of types from which they can choose.

Tasks that focus on procedural fluency

The most common tasks in textbooks are those that offer students opportunities to practice skills or procedures, being what Kilpatrick et al. (2001) described as procedural fluency. It is essential that mathematics teaching goes beyond this focus. Yet fluency, across many actions is indeed what students need to be very familiar with, so it is important that tasks that seek to develop fluency are chosen well and incorporated effectively into lessons.   It is common for mathematics teachers, especially from middle primary years onwards, to demonstrate specific procedures to their students, supplemented by repetitious practice of similarly constructed examples, the intent of which is to develop procedural fluency. This process is both boring and restrictive for students.

It is possible to learn about the processes of choosing good fluency tasks from considering alternative approaches to collaborative planning, commonly undertaken by Japanese teachers. The focus of Japanese mathematics lessons is often on the intensive study of particular examples, with students working on a single task for a whole lesson. This seems to have major advantages for the robustness of the mathematics learning, as is evident in the high standing of Japanese students in international comparative studies.

Fujii (2010) has analysed a range of school texts and research studies focusing on recommendations about what should be the first task posed to young children who are ready to move to subtraction involving numbers beyond 10. Fujii reported that in preparation for lessons with such a focus, Japanese teachers discuss among themselves characteristics of various examples such as whether 13 minus 5 offers more potential than 15 minus 7 for encouraging exploration of key ideas. In the lesson that results, the teachers pose only the task they have chosen. The intention is that students, after working on the task for themselves, will hear a range of strategies for completing such tasks devised by other students and then evaluate their own strategy against other suggested strategies. While this does not align with the ways that Australian teachers commonly utilise tasks that develop procedural fluency, choosing illustrative examples for detailed attention by the group could usefully be a focus of teacher planning. The advantage of doing this is that, rather than mindlessly following rules, students will come to see 'efficiency in strategy' as a matter for their conscious choice.

In considering which tasks will best foster fluency, teachers should also be looking to find optimal ways to incorporate tasks which develop fluency into lessons. Good et al. (1983), in their pivotal review of the teaching effectiveness literature, the importance of which has not lessened over time, described a lesson structure with the following sequence.

- After correcting homework, the teacher poses some old examples to check student facility with prerequisite skills.
- The teacher then presents some new examples, and asks students to complete some illustrative tasks.
- Next, further questions are posed in sets of similar complexity.
- Then the students' responses to set exercises are corrected.
- Some further examples are posed to the class to check both the students' accuracy and   their capacity to explain the process they used.
- Further examples are set for homework.

This sequenced structure is more likely to enhance the flow of lessons focusing on developing procedural fluency than the mere setting of examples for practice. The structure also has the potential to enhance conceptual understanding and develop some adaptive reasoning as well.

Tasks using models or representations that engage students

There has been substantial and sustained interest in Australia in tasks and lessons using interesting models or representations that both illustrate key mathematical principles and which have potential to engage students. Barbara

Clarke (2009) described such tasks as ‘representational tasks’, ones that are: ….. *explicitly-focussed experiences that engage children in developing and consolidating mathematical understanding*

The category of tasks that she described were intended to present physical or other representations that made abstract mathematics more tangible for students.

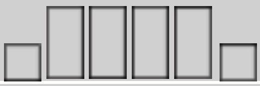
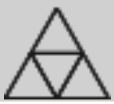
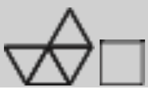
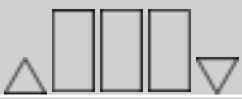
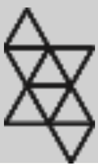
In his work, which built on an extensive program of research and development of tasks at the Shell Centre in the United Kingdom, Malcolm Swan (2005) encouraged teachers to ‘use rich collaborative tasks’. According to Swan, these tasks are ones that:

- emphasise methods rather than answers
- facilitate connections between topics
- support cooperative group work
- build on what the students bring to sessions
- explore common misconceptions.

An illustrative task using representations

The following is an example of such a task. This task is suitable for students from about Year 5. In it, the teacher provides students with a shuffled set of cards about one or several different polyhedra. The full set has cards for five different polyhedra, a total of 20 cards. For each polyhedron, there are four cards to that set – a name card, one with a representation of the net of the specific polyhedron, and two cards that refer to its properties of faces, edges and vertices. The task is for students to identify the polyhedra under discussion, by selecting the four descriptive and representational cards that match that form. The intention is that students will imagine and describe what each polyhedron would look like, but it is also possible to include photographs or actual models of the polyhedra.

Readers are reminded that each square would be cut up into a small card and presented to students for them to assemble and allocate, according to polyhedra form. The pedagogic idea here is that the students will work in small groups to sort the cards with instructions about requirements for them to explain their thinking as they match cards that are different representations or properties of particular polyhedra.

I am a rectangular prism	*My net is* 	*I have 6 faces and 8 vertices*
I am a tetrahedron	*My net is* 	*I have 4 faces and 6 edges*
I am square pyramid	*My net is* 	*I have 8 edges and 5 vertices*
I am a triangular prism	*My net is* 	*I have 6 vertices and 9 edges*
I am an octahedron	*My net is* 	*I have 8 faces and 6 vertices*

Such tasks can assist students in finding, clarifying and using appropriate language, they can provide a focus on different representations of the same idea, and their ‘solution’ can indicate to teachers what the students know. Such tasks are ideal for building conceptual understanding, and are readily adaptable to working in the same way in many other content domains in mathematics.

Contextualised practical problems

The use of contexts to situate mathematical problems is common internationally. Raffaella Borasi (1986), for example, defined 'context' as the situation in which a problem is embedded, providing problem solvers with information that may assist them in solving the problem. Meyer, Dekker and Querelle (2001), suggested that contexts can be used to motivate, can illustrate potential applications, can be a source of opportunities for mathematical reasoning and thinking, and can anchor student understanding.

In studying the classroom work samples and test responses of 273 children in six Year 4 classrooms and six Year 6 classrooms in the United States of America, Wiest (2001) found that the context of problems affected learning. A range of variables affected included students' interest in, and therefore their attentiveness and willingness to engage with problems; the strategies they used; the effort they expended; their perception of the difficulty of the task and their success in solving it; and the extent to which measurable learning outcomes were attained.

In its policy directions report on curriculum and evaluation standards the National Council of Teachers of Mathematics (1989), the peak professional body representing mathematics teachers in the United States of America, argued that problems using contexts enrich the experience of students learning mathematics. Brinker-Kent (2000) studied the use of mathematical tasks, set in contexts that were meaningful to the students in a culturally diverse elementary school, and concluded that all students are capable of learning significant concepts when they have the opportunity to explore the ideas in contexts that are meaningful to them.

In reporting on a study of teacher development in, and the classroom implementation of, a range of types of tasks, 'contextualised practical problems' were described by Clarke and Roche (2009) as occurring when the teacher situates mathematics within a realistic context to engage the students, with the motive of using the context as a stimulus for learning the mathematics.

The following task was developed by Doug Clarke and Anne Roche as part of an interactive student assessment intended for students in the upper primary and junior secondary years. They used attractive images of realistic cards, although the task is presented here as text.
If one pre-paid card for downloading music offers 16 songs for $24, and another offers 12 songs for $20, which is the better buy?

Tasks such as this one address both practical and specialised mathematical goals. The task is practical in that using pre-paid cards to purchase is a realistic context for students and so the context and the task would be familiar to them. The task also addresses an important application of ratios and rates (that of 'best buys'). There is a diversity of mathematical strategies that can be used to solve this task. These diverse strategies range from unitary methods (either comparing the number of songs per dollar, or cost per song), the common comparison methods (either the number of songs for $120, or cost of 48 songs), comparison of change method (4 extra songs for $4 more), and so on.

The point is that each of these strategies allows the teacher an opportunity to name and emphasise specialised mathematical ideas, once they have arisen from the students' investigations. Such tasks offer substantial potential to develop strategic competence and adaptive reasoning, are engaging for students, and are suited to collaborative activity.

Open-ended tasks

Various researchers have found that dealing with tasks or problems that have many possible solutions contributes to learning. Christiansen and Walther (1986) argued that tasks with open goals (that is many possible solutions) can engage students in productive exploration, and proposed that such tasks enhance motivation through increasing the students' sense of control. There are many types of open-ended tasks and the following elaborates just two types: investigations; and content specific tasks.

Investigations

The following example of an investigation type of task is an adaption of work by some of these researchers.

> *Collect some sports balls, such as a basketball, a baseball, a table tennis ball, and tennis ball.*
>
> *Describe these balls.*

The intent of this task is that students will define the properties (such as dimensions, mass, texture) of the balls on which they will focus, and then find ways to both describe the individual balls and compare the characteristics of the balls. It requires students to make choices, describe, measure, record, explain, and justify, which constitute some of our desired mathematical actions.

This sport balls task is also similar to the 'rich tasks' proposed as cross-disciplinary investigations by the Department of Education and Training, Queensland. The following is a description of the learning involved in such a rich task, labelled Pi in the sky. It was explained for teachers as follows:

Students will demonstrate an understanding of different mathematical approaches used to frame and answer questions about astronomy asked by cultures from three different historical ages. For each culture, they will immerse themselves in one such question as well as the ways in which the culture used or developed mathematics to frame and answer the question.

While such tasks present a kaleidoscope of options, which have potential to enable considerable creative learning, they are also extremely difficult for teachers to implement and for students to navigate. There are a number of challenges for teachers and students in seeking to solve such investigative tasks. First, there is substantial extraneous information that must be processed. Second, because such tasks have several different mathematical elements, it is hard for teachers to align such tasks with a curriculum that is sequential and already crowded. Third, it is difficult for students to know what they are meant to be learning. Finally, the lack of clarity in the focus makes the task of teaching difficult. While there are reports of teachers implementing such tasks effectively, often in a cross-disciplinary collegial culture, understanding and implementing such tasks also present challenges, which explains their limited uptake by teachers.

Content specific open-ended tasks

A similar approach, one that retains most of the benefits associated with such investigations, but which is more manageable for students and teachers, is one that involves open-ended tasks which are aligned with a sequential and topic specific curriculum. One way to do this is with what are described as content specific open-ended tasks. Some examples of such tasks are as follows:

a. Draw some letters of the alphabet on squared paper so that each letter has an area of 10 square units.

b. A commercial vegetable garden which has the shape of an 'L' has an area of 1 hectare. What might be the perimeter?

c. On squared paper, draw as many different parallelograms as you can with an area of 12 square units.

Each of these tasks addresses a specific aspect of the curriculum, ranging respectively from calculating area as counting squares to forming composite shapes, to developing a generalised rule for calculating area.

Each has multiple possible solutions and solution strategies and will therefore encourage and allow rich classroom discussions and create the expectation that students will explain what they have done and their underlying thinking. For example, in the case of the second task, because it does not need to be solved by the application of a taught routine, the students can make choices on the size of the parts of the 'L', and therefore expect to be invited to explain those choices. Those explanations provide teachers with important insights into the thinking of their students.

The tasks are accessible for most students since they can respond to the task by building on their current knowledge and understanding. In the first example above, some students might respond by drawing simple letters (L, C, T), whereas others might use more complicated letters involving many half squares (Z, K, R).

The tasks are engaging for students because they choose the level at which they engage. The tasks promote different types of thinking because there are many solution strategies possible, and different ways of thinking, since responses can be represented in different ways.

There are a number of pedagogic matters to be considered in relation to teaching open-ended tasks. One advantage of such tasks is that they are readily adaptable for students who experience difficulty, and can easily be extended for students who finish quickly. Another advantage is that devising them is facilitated by collegial action between teachers. A further issue is that the classroom climate needs to be managed so that students feel free to offer their ideas on possible options.

To further illustrate the ways that such tasks might contribute to learning, the following discussion deals with issues which often arise, on Figure 2c (drawing different parallelograms of a given area). In classroom trials, students working on this task have been prompted to query their conceptual understandings. They commonly ask questions about:

- whether rectangles and squares are also parallelograms. This leads to a class discussion of the inclusiveness and parsimony of those definitions.
- which parallelograms can be treated as the same and which as different (that is, whether a parallelogram with a base of 4 and a height of 3 is the same as one with a base of 3 and a height of 4, if their angles are the same). This query allows discussion of the concepts of transformations, and congruence.
- whether there are only two parallelograms with a base of 4 and a height of 3. Most students are able to find the shape with a base of 4 and with the side going along the diagonal of the squares on the paper, and, with prompting, also find the rectangle. But many have difficulty finding others. This allows discussion of the key issue that the area of the parallelogram is determined by the base and height and not by the interior angles or the length of the side. Therefore, there can be multiple such parallelograms!

These concepts are important mathematical ideas, itemised in many Australian mathematics curricula. By using open-ended tasks, teachers can facilitate discussion of such ideas, thus giving students an opportunity to develop conceptual clarification and proficiency in mathematical reasoning. This pedagogy models adaptive application of mathematical concepts and processes which will be important to these students in their learning. Each of these key mathematical issues arises as an outcome of the students' explorations, and so such tasks allow teachers to emphasise all five mathematical actions: conceptual understanding. procedural fluency, strategic competence, adative reasoning and productive disposition.

Constraints on use of tasks

A first step in addressing constraints is awareness of them. One of the major constraints that teachers experience when utilising such tasks is that many students avoid risk taking and do not persist with the challenges that are required in order to complete the task. And teachers are sometimes complicit in this avoidance strategy.

Desforges and Cockburn (1987), for example, reported on a detailed study of primary classrooms in the United Kingdom and found that students and teachers conspired with each other to reduce the level of risk for the students. They argued that teachers can sometimes avoid the challenge of dealing with students who have given up, by reducing the demand of the task.

Stein in a detailed classroom-based study of task implementation, also noted the tendency of teachers to reduce the level of potential demand of tasks for some students. He argued that teachers sometimes modify tasks at the planning stage if they anticipate that students cannot engage with the tasks without considerable assistance, and also once they see students not responding as intended. It is important for teachers to be aware of this tendency and, if they note it in themselves, to develop strategies to overcome it.

After recognising the role of the teacher in creating an optimal learning environment for all students, the next step is establishing a classroom culture that builds community, encourages effort and acceptance of errors, and not only tolerates, but celebrates, difference. Such a classroom culture can be established through explicit norms.

Cobb and McClain (1999) used the term 'mathematical norms' to describe mathematical tasks and their possible trajectories, the mathematical actions which are to be valued in the learning, and all and any of the products which students contribute to the learning in classes. Complementing this notion is the concept of socio-mathematical norms, which includes the modes of communication, types of responses valued, and expectations about risk taking and tolerance of others' errors. One of the roles of the teacher is to establish the norms that operate in the classroom so that the type of task use described in this section is not subverted deliberately or inadvertently by the actions of some students. A necessary prerequisite to implementing the type of teaching based on representational, contextual and open-ended tasks described above is the establishment of the appropriate classroom culture.

Problem posing

One strategy that may be useful for encouraging students to engage with problems and to persist, even if challenged, is described as 'problem-posing'. Leung (1998) and English (2006) both proposed problem posing as a process in which students not only reformulate problems with which they are presented, but they argued students should also pose their own problems, either for themselves or for others in the class to solve. One effect is that students generally pose problems at the level with which they are most comfortable, and the teacher's challenge is then one of how to move them into a less comfortable stage by proposing more complex tasks, possibly with other students. Another effect of problem posing is that many of the above tasks require students to ask themselves questions, even when they are working on a specific task, and so a willingness to ask questions about what is possible assists students in exploring the potential of many tasks.

This capacity to pose questions is one of the goals of mathematics teaching, and it has uses in adult life too. It is the essence of what Kilpatrick et al. (2001) described as 'adaptive reasoning', and has important elements of strategic competence.

Seeking students' opinions about tasks

In this author's presentation at the Teaching Mathematics? Make it count conference, he reported on a project that sought insights into aspects of task use by examining student preferences (Sullivan, 2010). The project drew on earlier research on students' attitudes and their beliefs about mathematics and its learning, and the value they ascribe to learning mathematics. Sullivan outlined how he, with colleagues D. Clarke and B. Clarke, surveyed students from 95 Year 5 to Year 8 classes, inviting them to compare different types of tasks and to indicate their preferences for the tasks they liked doing and also those from which they felt they best learn.

The researchers found that students have a wide diversity of preferences for the types of tasks that they enjoy and also for the types of tasks from which they think they can best learn. Most significantly, the students' preferences for particular types of tasks were not dependent on whether they were confident in their own ability or whether they reported positive attitudes to learning mathematics. Sullivan (2010) concluded that the diversity of student preferences make it essential that teachers incorporate a variety of task types into their planning and teaching, so that they 'reach' all their students, and also that they explain the purpose of each type of task to the students, so students can more readily recognise the type of task it is.

Concluding comments

This section has argued that choosing appropriate tasks is one of the key decisions for teachers, and has presented a range of possible types of tasks, all of which can make a positive contribution to different aspects of student learning. It has been established also that having students pose questions in their own words allows for explicit articulation of mathematical learning, and for the understanding that there may be multiple ways of solving a problem. Recognising the value of these mathematical learning outcomes, both during the school years and in later life, research suggests that mathematics teachers should incorporate most of these types of tasks into the planning of lesson sequences, and during individual lessons. Indeed, one of the expectations in The Shape of the Australian Curriculum: Mathematics (ACARA, 2010a) is that teachers will use a range of types of tasks that allow students opportunities to solve problems, explain their reasoning, and build their understanding, as well as developing the necessary fluency.

Of course using challenging tasks creates its own expectations for teachers, including the need to maintain the demand of tasks and to support students in persisting. All of these aspects of task choice and use can productively be the focus of teacher learning initiatives.