The 5 Principles aren't just for students who are at present high achievers or for those who have already mastered the basics. A problem-solving approach focused on process instead of content is essential to helping struggling learners as well. Boykin and Noguera (2011), in their book *Creating the Opportunity to Learn*, also list many of these principles in their description of classroom environments which effectively eliminate racial and ethnic achievement gaps. No matter who you teach or where, the 5 Principles framework can help you improve learning for your students.

DEPTH AND RIGOR

Consider the following:

> Miguel collects baseball cards. Last week he had 217 cards in his collection. Today, his aunt gave him two dozen more for his birthday. How many cards does he have now?

This is a common type of problem seen in elementary-level textbooks. You might find it at the end of a worksheet about adding multi-digit numbers. The process for solving is straightforward, though it does require a few mental steps to complete it successfully.

Project Mentoring Mathematical Minds (Project M^3) is a research-based program designed to challenge and motivate mathematically talented students in Grades 3–6. Now compare the baseball card problem with this Project M^3 problem adapted from Lesson 2, *"Card Game Capers"* from the book *Mystery of the Moli Stone* (Gavin, Chapin, Dailey, & Sheffield, 2006):

> You and your friends are going to play a game using a set of cards numbered from 0 to 9. On your turn, you are going to draw three cards from the facedown deck, one at a time. The object is to make the largest 2-digit number you can using your cards, with the leftover card being discarded. The catch is that you must decide where to write each digit before you draw the next: tens place, ones place, or discard. If you draw a 4 as your first card, where should you write it, and why?

This, too, is a problem, but it seems different in some fundamental ways. As adults, we can quickly see the correct path to the solution of the baseball card problem, and finding that answer is just a matter of working our way through that path. The card game question, on the other hand, feels fuzzier.

The first example was designed to apply one specific, abstract mathematical skill to a concrete situation which might occur in the real world.

The second, however, is designed to apply various ways of reasoning and conceptualizing about numbers and their relationships.

Most of what passes for "problems" in available math resources are of the first variety. They aren't actually problems so much as they are exercises. Wikipedia defines an exercise as "a routine application of . . . mathematics to a stated challenge. Teachers assign mathematical exercises to develop the skills of their students" (Exercise [mathematics], 2014, para. 1). The key word here is *routine*. Like physical and artistic exercises, they are intended to be repeated frequently until a particular process becomes fluent and automatic. Mathematical exercises are the cognitive equivalent of scales in music, ball handling in sports, and knife skills for a chef. Important for the practitioner to do, certainly, but they don't constitute a complete performance.

Mathematical exercises are the cognitive equivalent of scales in music, ball handling in sports, and knife skills for a chef. Important for the practitioner to do, certainly, but they don't constitute a complete performance.

We may think that it's enough to have students complete more story problems. But, Resnick (1988) points out

> Story problems do not effectively simulate out-of-school contexts in which mathematics is used . . . [T]he language of story problems is highly specialized . . . requiring special linguistic knowledge and distinct effort on the part of the student to build a representation of the situation described. Furthermore, this representation, once built, is a stripped down and highly schematic one that does not share the material and contextual cues of a real situation. (p. 56)

Instead of giving students real problems to solve, we've just given them yet another set of mathematical symbols to manipulate. This is far from the depth of understanding that we want.

The word "rigor" is hard to avoid today, and it provokes strong reactions from educators. Policymakers tout its importance, and publishers promote it as a feature of their materials. But, some teachers share the view of Joanne Yatvin, past president of the National Council for Teachers of English. To them, rigor simply means more work, harder books, and longer school days. "None of these things is what I want for students at any level" (Yatvin, 2012, para. 3).

We have adopted jargon without clearly understanding it. "'People don't know what it means,' said longtime educator and consultant Barbara Blackburn. 'The teachers I work with are being told they're supposed to include rigor. It's certainly the flavor of the month. But teachers all say everyone is telling me what to do but they can't tell me how to do it'" (Colvin & Jacobs, 2010, para 20). For classroom teachers, then, the more important question is one of practice: How do we create rich environments where all students learn at a high level? One useful tool, Norman Webb's (2005) Depth of Knowledge Levels, can help teachers meet that challenge. Depth of Knowledge (DOK) categorizes tasks according to the complexity of thinking required to successfully complete them.

Level 1. Recall and Reproduction. Tasks at this level require recall of facts or rote application of simple procedures. The task does not require any cognitive effort beyond remembering the right response or formula. Copying, computing, defining, and recognizing are typical Level 1 tasks. Recall of basic math facts and application of memorized algorithms would be math tasks at this Level.

Level 2. Skills and Concepts. At this level, a student must make some decisions about his or her approach. Tasks with more than one mental step, such as comparing, organizing, and estimating are usually Level 2. Math examples would include when a student has to select from among several possible well-defined paths or algorithms, or has to use an algorithm in an unconventional, but straightforward way. The baseball card problem in the last section is an example of a Level 2 task, since the student must first determine the numeric value of "two dozen," then select and apply the correct algorithm.

Level 3. Strategic Thinking. At this level of complexity, students must use planning and evidence, and thinking is more abstract. A task with multiple valid responses where students must justify their choices would be Level 3. Examples include designing an experiment, or analyzing characteristics of a genre. In mathematics, solving a non-routine problem, or explaining the reasoning behind a Level 2 application would be examples of Level 3 tasks. The number-card game in the previous section is a good example of a Level 3 task.

Level 4. Extended Thinking. Level 4 tasks require the most complex cognitive effort. Students synthesize information from multiple sources, often over an extended period of time, or transfer knowledge from one domain to solve problems in another. Designing a survey and interpreting the results, analyzing multiple texts to extract themes, or writing an original myth in an ancient style would all be examples of Level 4. A Level 4 math task would involve multiple sources of raw data, or complex problems requiring innovative thinking with no routine solution path.

> DOK levels are not developmental. All students, including the youngest preschoolers, are capable of strategic and extended thinking tasks . . . All students should have opportunities to do complex reasoning with advanced content . . . even Kindergarteners.

You may be asking at this point, "Well, what is a reasonable distribution? How often should I be doing tasks at each level? What's the right sequence?"

DOK levels are not sequential. Students need not fully master content with Level 1 tasks before doing Level 2 tasks. In fact, giving students an intriguing Level 3 task can provide context and motivation for engaging in the more routine learning at Levels 1 and 2.

DOK levels are also not developmental. All students, including the youngest preschoolers, are capable of strategic and extended thinking tasks. What those tasks look like will differ. Tasks that require Level 3 reasoning for a fifth grader may be a Level 1 or 2 task for a high school student who has learned more sophisticated techniques. All students should have opportunities to do complex reasoning with advanced content. Recent research strongly supports this, even for Kindergarteners. "All children, regardless of their early childhood care experiences, benefit from more exposure to advanced mathematics content" (Claessens, Engel, & Curran, 2014, p. 426; see also Engel, Claessens, & Finch, 2013). To find the right balance, ask yourself these questions: "What kinds of thinking do I want students to routinely accomplish? If my own children were participating, what would I want them to be doing? What's the most effective way to spend the limited classroom time I have?" You should decide how often you focus on tasks at each level so students gain the most from the learning opportunities you design.

Regardless of how you define "rigor," the important thing is that students are thinking deeply on a daily basis. Webb's Depth of Knowledge outline gives us a framework and common language to achieve that effect in your classroom.

Distinguishing Between DOK Levels in Math

Math specialist and teacher Robert Kaplinsky has developed a tool to help teachers recognize the differences between Depth of Knowledge levels in mathematics tasks. The tool, available at http://robertkaplinsky.com/tool-to-distinguish-between-depth-of-knowledge-levels/, provides explicit examples of problems for all grade levels illustrating DOK levels 1, 2, and 3. Figure 2.1 gives a selection of these examples. Try each of the problems yourself so you can experience the kind of thinking needed to solve each level. In particular, notice that Levels 2 and 3 require a more sophisticated understanding of the concept and cannot be solved through a rote or routine process.

Now try creating your own set of problems for a topic you are teaching soon. Begin with a DOK 1 exercise like one you might find in your math textbook. Then develop problems of increasing depth that require more complex application. There is no formula for this process: keep referring back to the DOK levels, and share and discuss the problems you've created with colleagues or students.

Figure 2.1 Kaplinsky's (2015) Tool to Distinguish Between DOK Levels

Topic	**Area and Perimeter**	**Probability**	**Quadratics in Vertex Form**
CCSS Standard(s)	3.MD.8 4.MD.3	7.SP.5 7.SP.7	F-IF.7a
DOK 1 Example	Find the perimeter of a rectangle that measures 4 units by 8 units.	What is the probability of rolling a 5 using two standard 6-sided dice?	Find the roots and maximum of the quadratic equation below: $y = 3(x-4)^2 - 3$
DOK 2 Example	List the measurements of 3 different rectangles that each has a perimeter of 20 units.	What value or values have a $\frac{1}{12}$ probability of being rolled using two standard 6-sided dice?	Create three equations for quadratics in vertex form which have roots 3 and 5, but have different maximum and/or minimum values.
DOK 3 Example	What is the greatest area you can make with a rectangle that has a perimeter of 24 units?	Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each: Rolling a sum of ____ on two ____-sided dice is the same probability as rolling a sum of ____ on two ____-sided dice.	Create a quadratic equation using the template below with the largest maximum value using the whole numbers 1 through 9, no more than one time each: $y = -\square(x - \square)^2 + \square$

5 Principles of the MODERN Mathematics Classroom

Creating a Culture of Innovative Thinking

GERALD AUNGST
Foreword by Mark Barnes

CM CORWIN MATHEMATICS