PRE-ROUND READING A: MAKING LEARNING VISIBLE THROUGH APPROPRIATE MATHEMATICAL TASKS

BACKGROUND INFORMATION

John Hattie's books are based on three phases of learning: surface learning; deep learning; and transfer Learning. These extracts are provided to help make the model clearer.

Almost everything in published research works at least some time with some students. Our challenge as a profession is to be more precise in what we do and when we do it. Timing is everything, and the wrong practice at the wrong time undermines efforts. Knowing when and how to help a student move from sufficient levels of surface learning to deep learning is one of the hallmarks of expert teachers

Surface Learning

It is easy to assume that surface learning means "superficial" or "shallow" or that by surface-level learning we mean rote memorization of procedures and vocabulary that have been traditionally taught at the beginning of the lesson and are disconnected from conceptual understanding. This is not what we mean by surface learning. Rather, the phrase surface learning represents an essential part of learning made up of both conceptual exploration and learning vocabulary and procedural skill that give structure to ideas.

Surface learning teaching strategies for Mathematics include: mathematical talk; daily number talks; guided questioning; worked examples; direct instruction (ie short portions of maths lessons where teachers provide more information); strategic vocabulary instruction; word walls; manipulatives; and spaced practice.

Deep Learning

Deep learning focuses on recognising relationships among ideas. During learning, students engage more accurately and deliberately with information in order to discover and understand the underlying mathematical structure. Students who are involved in deep learning are:

- *displaying, explaining, and justifying mathematical ideas and arguments*
- *communicating*
- *reasoning, and*
- *analysing mathematical relationships and connection*

One teaching strategy for Maths that supports deepen learning is accountable talk which sets discourse expectations to enrich maths discussions. Others are encouraging students to use multiple representations and the strategic use of manipulatives.

Transfer Learning

All the work teachers do is for naught if students fail to transfer their learning appropriately by applying what they have learned in new situations. Hattie explains that transfer learning is about the ways in which students construct knowledge and reality for themselves as a consequence of surface knowledge & deep understanding.

The concern is that mathematics instruction too often stops at the surface level of learning, and students (particularly struggling students) either fail to go deep and transfer, or they transfer without detecting similarities and differences between phenomena. When this happens, the transfer does not make sense, and too often students see this as evidence that they can't do mathematics.

Transfer is both a goal of learning and also a mechanism for propelling learning to the next level. Transfer as a goal means that teachers want students to begin to take the reins of their own learning, think metacognitively, and apply what they know to real-world contexts. It also prepares them to move through the progression of mathematical understanding as ideas build on each other across grade levels. It's when students reach into their toolbox and decide what tools to employ to solve new and complex problems on their own. When students reach this phase, learning has been accomplished.

PRE-READING A:

MAKING LEARNING VISIBLE THROUGH APPROPRIATE MATHEMATICAL TASKS

Extract from: *Visible Learning for Mathematics: What Works Best to Optimize Student Learning*

John Hattie, Douglas Fisher and Nancy Frey, Corwin 2017

Ms. Clark was planning a lesson on counting the value of coins for her first graders. Her learning intention for the lesson was for students to determine the value of up to four coins including pennies, nickels, and dimes. Her success criterion was for students to successfully apply their understanding to a new situation. She considered the work in the first-grade text that included drawings of several coins of which students were to determine the total value. Since they had been spending a lot of time on this skill, she was certain this would not be very challenging for her students. Instead, she decided to give them the following task.

> You are going to the store and you want to buy a banana that costs 25¢. You have lots of pennies, nickels, and dimes. What coins can you use to pay for the banana?

Ms. Clark brought a variety of coins to class so that each group had a selection of coins to help them with the problem. She was surprised at the reluctance of the students to get started on what she thought would be an enjoyable task. It turned out some students didn't recognize the real coins (even though they recognized the drawings in the textbook). Other students recognized the coins but had no idea of how to put them together to make 25¢. Ms. Clark did not jump right in to tell the students what to do. Instead, she encouraged them to work in groups to support each other in solving the problem. She was intrigued to watch the groups form based on what students could do (recognize or count the coins). Soon, one group of students raised their hands to show their answer of two dimes and a nickel. When Ms. Clark asked if there was another way to make 25¢, the students were dumbfounded. They had never solved a problem with more than one correct answer! Interestingly, the students set to work to find other solutions, challenging themselves to find all of the possible combinations!

Making Learning Visible Through Appropriate Mathematical Tasks

The banana problem is an example of students having surface learning (recognizing coins and/or knowing the value of individual coins) and taking that learning to a more complex level through deep learning (combining the value of various coins) to transfer learning. Not only did they have to recognize and add the value of the coins, but unlike the textbook exercises, they also had to determine which coins to use. Giving students appropriate tasks at the right time in their learning cycle is crucial to move students from surface to deep and transfer learning.

Exercises Versus Problems

It is important to have a common understanding of the types of tasks we assign to students. **Exercises**, which typically make up most of traditional textbook practice, are provided for students to practice a particular skill, usually devoid of any context. Although these are casually referred to as problems, in reality they are simply practice exercises.

Exercises are meant to practice a particular skill, but are noncontextual.

Problems have contexts—they are usually written in words that can be situations that apply or provide a context for a mathematical concept. One category of problems is an application that focuses on the use of particular concepts or procedures. Another category of problems is nonroutine or open-ended problems that involve much more than applying a concept or procedure. We will explore each of these types of problems in more detail in the next section.

Problems are usually written in words that can be situations or provide a context for a mathematical concept.

There are a few items that we need to address before we more fully explore the types of tasks that are useful in various phases of learning in mathematics.

EFFECT SIZE FOR SPACED VERSUS MASS PRACTICE = 0.71

- Spaced practice—also known as distributed practice—is much more effective than mass practice. We will discuss this more in Chapter 4. In practical terms, this means that students should do a few exercises or problems on a given concept each day over several days rather than a lot of problems for only one or two days.
- Math is not a speed race. Teachers should be very careful with timed tests. Neither fluency nor stamina requires that students work as quickly as humanly possible. Giving students a test that requires them to speed through problems reinforces an idea that they should prioritize by doing the "easy" problems first and not spend valuable time on problems that require deeper thinking. Too often, timed tests or speed games are used to check for fluency with basic mathematics facts. The problem is, speed is not part of fluency. Fluency requires flexible, accurate, and efficient

thinking. Fluency also requires a level of conceptual understanding. One would not be considered fluent in a foreign language if he or she could speak it by mimicking without any comprehension! Students would be better served with practice developing fluency rather than racing through written tasks or activities. In addition, speed races also make some students believe that they are not good at math. The attitude students have toward mathematics is important and can impact their willingness to try.

- Tasks should not focus exclusively on procedures. Sara excelled in math at a young age. She seemed to understand numbers, and she was very good at learning a procedure and executing it repeatedly on her own. But she was never asked to explain why these procedures worked. Bring down the last number under the house when doing long division? Sure, why not. Why does that lead to a correct answer? How do I apply that skill to real-world situations if I don't understand what it means? Sara had no idea, and it didn't seem to matter to her teacher. This was a case of focusing on procedural skills and sacrificing conceptual understanding.

We are not arguing that students shouldn't learn long division. But we don't think that students gain much from doing long division mindlessly, either. The goal should be for students to develop a transferable and flexible understanding of processes like division, and they should have the opportunity to construct this understanding in a meaningful context. Doing extensive, repeated, context-free long division exercises is just not aligned to this goal.

Instead, students should be expected to engage in reasoning, exploration, flexible thinking, and making connections. They know that learning isn't easy, and they should enjoy the success of meeting the challenges that learning demands of them (Hattie, 2012). Students need deliberate practice, guided by the teacher, not repetitive skill-and-drill tasks. Some tasks should provide students an opportunity to engage in mathematical modeling—taking a problem or situation, representing it mathematically, and doing the mathematics to arrive at a sensible solution or to glean new information that wouldn't have been possible without the mathematics.

Still other tasks require that students practice applying a concept in different situations. To facilitate strategic thinking, some tasks should be open-ended and have multiple paths to get to the solution or, in some cases, solutions. Math tasks don't always have to be fun, but they can be interesting and useful.

Should students work on exercise sets, in which they develop skill in long division? Sure, but these types of tasks won't be discussed here for several reasons. First, we have seen that teachers are already quite good at assigning exercises from a textbook, and reading about this would be a waste of your time. More importantly, though, the research evidence suggests that application of a concept, in varying contexts or in ways that offer sense-making opportunities, is more effective in building true fluency than doing repeated, nearly identical manipulations of numbers (NCTM, 2014).

It is useful for students to be able to perform math operations flexibly and efficiently, as it frees up cognitive space to apply these operations to novel situations and relate these operations to other mathematics concepts. But in most mathematics classes, this type of automaticity tends to be emphasized way too early in the learning cycle. It also tends to take up a disproportionate amount of class time. Procedural fluency cannot be developed without true and meaningful comprehension, and "drill-and-kill" exercises without understanding can harm students' mathematical understandings, their motivation level, and the way they view mathematics. Students who learn procedures at the expense of mathematical thinking often fail to develop an understanding of what they're doing conceptually, and teachers find that it's more difficult to motivate students to really understand a concept if they can already execute a shortcut. What's needed is a restoration of the balance: A strong conceptual foundation makes fluency building more efficient, meaningful, and useful for students. So it really is worth devoting a lot more learning time to the conceptual understanding that undergirds procedural knowledge. Children need to learn the relationship between procedures and concepts in order to become increasingly fluent thinkers.

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Problems fall into two categories: applications and nonroutine problems. **Applications**—often called word problems or story problems—are problems, usually related to real-life experiences, in which students use or apply a mathematical concept or skill they have learned. Interestingly, these problems usually follow the exercises in a traditional textbook lesson. However, they should also be used to introduce an idea in order to allow students to model a situation and develop conceptual understanding, connect that understanding to procedural skills, and then practice that skill through more applications and exercises. For those familiar

Applications are problems, usually related to real-life experiences, in which students use or apply a mathematical concept or skill they have learned.

with Cognitively Guided Instruction (Carpenter et al., 2014), this is the pathway used in that philosophy. Application problems can range from straightforward (solution reached by applying well-practiced operations) to difficult (involving application of new ideas, several steps, and/or multiple representations).

Non-routine or complex problems are problems that involve more than applying a mathematical procedure for solution.

Non-routine or complex problems are problems that involve more than applying a mathematical procedure for solution. These types of problems are usually met with student reactions of "I don't know what to do!" because a simple procedure is not the pathway to a solution. Rather, students need to use a variety of strategies and some "out-of-the-box" thinking to solve these types of problems.

When we think about the kinds of mathematical tasks we want to use with our students, and when we should use each kind of task (and there is a place in mathematics instruction for each type of task), we need to think about what we want to achieve with the task. What are our learning intentions? What role does the task play in helping students meet the success criteria for the lesson?

In the next sections, we will examine two frameworks for classifying problems. One focuses on the level of difficulty/complexity of the task, and the other focuses on the kind of thinking required by the student. One is not better than the other, but given your own realm of experience, one may be more helpful than the other as you work to connect exercises/problems with surface, deep, and transfer learning. We will go into more detail with examples in future chapters. Our intention here is to get you familiar with the descriptions and the need for hard thinking about the kinds of tasks you assign to your students to make your teaching positively impact student learning.

Difficulty Versus Complexity

Difficulty is the amount of effort or work one must put in.

Complexity is the level of thinking, the number of steps, or the abstractness of the task.

In order to help students master all dimensions of rigor (conceptual understanding, procedural fluency, and applications) and to help students' progress toward owning their own learning and then transferring that learning to new situations, it is important for teachers to think carefully about the level and type of challenge a given task provides. Unfortunately, some people confuse difficulty with complexity. We think of **difficulty** as the amount of effort or work a student is expected to put forth, whereas **complexity** is the level of thinking,

DIFFICULTY AND COMPLEXITY

Figure 3.1

the number of steps, or the abstractness of the task. We don't believe that teachers can radically impact student learning by simply increasing the volume of work. We know that students learn more when they are engaged in deeper thinking. Figure 3.1 shows how we think of this in four quadrants.

The fluency quadrant that includes tasks of low difficulty and low complexity is not unimportant; it's where automaticity resides. For example, once students have mastered conceptual understanding of addition and subtraction (what do they mean and what do they look like?) and learned thinking strategies and procedures for computing sums and differences, they need to build fluency so that they are flexible, accurate, and efficient with these operations. Students should be able to do basic mathematical calculations quickly and effortlessly in order to free up the cognitive space to connect the operations to more complex examples or to larger concepts. There are times when you will want students to build automaticity on certain types of procedures. Instant retrieval of basic number facts is foundational for being able to think conceptually about more complex mathematical tasks. Hattie and Yates (2014) assert that these retrievals are the product of

> a combination of exposure to others, working it out for yourself, playing with concrete materials, experimenting with different forms of representation, and then rehearsing the acquired knowledge unit within your immediate memory, transferring it into long-term memory, and having it validated thousands of times. (p. 57)

Video 3.1
What We Mean by Tasks With Rigor

http://resources.corwin.com/VL-mathematics

If students' mathematical experiences are limited to this quadrant, learning isn't going to be robust. The stamina quadrant—high difficulty but low complexity—is where tasks that build perseverance reside. Stamina refers to the idea of sticking with a problem or task even when the work is difficult and requires patience and tenacity. This type of task would be a problem or exercise (yes, they both have a place here) in which students are taking their current knowledge and extending it to a more difficult situation. The first-grade banana task that opened this chapter is a good example of a task that promotes stamina. Students were able to complete earlier work with counting coins in the textbook examples, but they needed to apply this knowledge differently and think strategically about the different ways to find *all* of the possible solutions, and then justify how they knew they had them all.

The daily practice of having students work independently to resolve a problem before consulting peers is one example of helping to build stamina, as it draws on the learner's capacity to stick with a problem. Add to that the additional step of consulting one another and then returning to the problem individually a second time to make any corrections, and now you're extending their stamina even further.

The strategic thinking quadrant addresses tasks that have a lower level of difficulty, but a higher degree of complexity. Some rich mathematical tasks fall into this category, as they draw on students' ability to think strategically. An example of strategic thinking is having students connect their understanding of division of whole numbers to division of decimals before any specific procedure is explored. In this task, students must think about what they know about division and what they know about decimals to make conjectures about place value in the quotient.

Mr. Beams has a very strange calculator. It works just fine until he presses the = button. The decimal point doesn't appear in the answer. Use what you know about decimals and division to help him determine where the decimal point belongs in each quotient. Be ready to justify your thinking!

1. 68.64 ÷ 4.4 = 156
2. 400.14 ÷ 85.5 = 468
3. 0.735 ÷ 0.7 = 105
4. 51.1875 ÷ 1.05 = 4875

This task requires students to extend their understanding from previous learning to situations that are much more complex. Complexity is often supported by having students work in groups and justify their thinking. Students will likely be stretched to consider how to resolve problems collaboratively, attend to group communication and planning, and monitor their own thinking and understanding.

The final quadrant, which describes expertise, includes those tasks that are both complex and difficult. These tasks, in one form or another, push students to stretch and extend their learning. A favorite task for fifth or sixth graders is the Handshake Problem, which includes both complexity and difficulty.

Twenty-five people attended a party. If each person shakes hands with every other person at the party, how many handshakes will there be?

This problem can be pretty overwhelming as there is not a particular process or operation that will lead to a solution. Rather, students might work together to use a combination of problem-solving strategies to get started, including acting it out, looking for a pattern, making a table, or starting with a simpler problem. What makes this problem even more interesting (and complex) is the opportunity for students to make a generalization (find a rule) so they can determine the number of handshakes for any number of guests—even 1,000!

This is certainly not an exhaustive list; rather, it is meant to be illustrative. As part of each lesson, teachers should know the level of difficulty and complexity they are expecting of students. They can then make decisions about differentiation and instructional support, as well as feedback that will move learning forward.

Students need regular contact with tasks that allow them to explore, resolve problems, and notice their own thinking. They need tasks that present the right amount of challenge relative to their current performance and understanding, and to the success criteria deriving from the learning intention. Teachers should select tasks that help students push their thinking, but are not so difficult that the learner sees the goal as unattainable. Teachers *and* students must be able to see a pathway to attaining the goal. This supports the second effective teaching practice in NCTM's *Principles to Actions*: Implement tasks that promote reasoning and problem solving. The tasks that teachers assign must

1. Align with the learning intention.
2. Provide students an opportunity to engage in exploration and make sense of important mathematics.
3. Encourage students to use procedures in ways that are connected to understanding.
4. Provide students opportunities to implement the standards for mathematical practice.
5. Allow teachers and students to determine if the success criteria have been met.

This is why relating a task to prior learning is so important (Hattie, 2012).

A Taxonomy of Tasks Based on Cognitive Demand

A second framework for thinking about how to strategically select mathematical tasks aligned to learning intentions and success criteria is one that presents a taxonomy of mathematical tasks based on the level of cognitive demand each requires (Smith & Stein, 1998). **Cognitive demand** is the kind and level of thinking required of students in order to successfully engage with and solve the task (Stein, Smith, Henningsen, & Silver, 2000).

Cognitive demand is the kind and level of thinking required of students in order to successfully engage with and solve a task.

This taxonomy has been embraced by the National Council of Teachers of Mathematics (NCTM, 2014) for good reason, as it provides a powerful

CHARACTERISTICS OF MATHEMATICAL TASKS AT FOUR LEVELS OF COGNITIVE DEMAND

Levels of Demands

Lower-Level Demands (Memorization)

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous; such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-Level Demands (Procedures Without Connections)

- Are algorithmic; use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task
- Require limited cognitive demand for successful completion; little ambiguity exists about what needs to be done and how to do it
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of on developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-Level Demands (Procedures With Connections)

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations; making connections among multiple representations helps develop meaning
- Require some degree of cognitive effort; although general procedures may be followed, they cannot be followed mindlessly—students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding

(Continued)

(Continued)

Higher-Level Demands (Doing Mathematics)

- Require complex and non-algorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

Source: Smith and Stein (1998). Used with permission.

Note: These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level-thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM, 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and Henningsen, 1996; Stein, Lane, and Silver, 1996).

Figure 3.2

structure to types and characteristics of mathematical tasks, providing teachers with criteria that enable them to align the type of task they choose with the learning intention and success criteria for a given outcome (see Figure 3.2).

Traditionally, the majority of classroom instructional time is spent on tasks with lower level cognitive demands that require memorization and/or procedures without connections. These are not bad tasks, and there is a time and place for them, but they do not provide students the range of learning experiences they need to develop mathematical habits of mind, such as looking for patterns and using alternate representations (Levasseur & Cuoco, 2003). Memorization tasks that follow the development of conceptual understanding facilitate learning at the surface level. And surface learning is important and should not be minimized. There has been much misdirected criticism of surface learning because it is often confused with shallow learning. That said, too much emphasis on surface learning at the expense of learning that deepens over time and transfers to new and novel situations does not provide students with true mathematical experiences. Balance is warranted.

Tasks with higher levels of cognitive demand on Smith and Stein's taxonomy—those that connect procedures to understanding—require students to understand relationships between concepts and processes as they analyze and explore the task and its parameters. But the process doesn't stop there. Tasks that call for higher level cognitive demand extend even further to those requiring more complex thinking. There is no predictable or well-rehearsed pathway (algorithm) that is suggested by the task, or by a similar and already-worked example. Tasks such as these provide students an opportunity to engage transfer learning.

However, effective teachers don't leave these things to chance. Instead, they provide problem-solving experiences in which students engage with rich tasks that require them to mobilize their knowledge and skills in new ways. A close association between a previously learned task and a novel situation is necessary for promoting transfer of learning. In time, these become tasks that stretch students' problem-solving abilities as they self-monitor and self-regulate their learning. This is transfer learning in action.

EFFECT SIZE FOR PROBLEM-SOLVING TEACHING = 0.61

Whether you are looking at a task in terms of difficulty versus complexity or the level of cognitive demand students must employ, appropriately challenging tasks may produce some level of student anxiety when they are first introduced. As we have noted before, that's okay, because students should expect learning to require an effort as they grow to appreciate cognitively demanding tasks. An often-needed requirement for learning to occur is some form of tension, some realization of "not knowing," a commitment to want to know and understand—or, as Piaget called it, some "state of disequilibrium" (Hattie, 2012).When students are assigned rich tasks, they use a variety of skills and ask themselves questions, make meaning of mathematics, and ultimately build a healthy and realistic relationship to mathematics as something that is engaging, interesting, and useful—and something that makes sense.

Figure 3.3 includes examples of mathematical tasks for each level of cognitive demand.

We will refer back to these tasks and present additional tasks for your consideration in the coming chapters. In the meantime, we encourage you to sharpen your pencils and experience the levels of cognitive demand along with some metacognition by completing these tasks. Note that answers are not provided in the back of this book!

EXAMPLES OF TASKS AT EACH OF THE FOUR LEVELS OF COGNITIVE DEMAND

Lower-Level Demands

Memorization

What is the rule for multiplying fractions?

Expected student response:

You multiply the numerator times the numerator and the denominator times the denominator.

or

You multiply the two top numbers and then the two bottom numbers.

Procedures Without Connections

Multiply:

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{5}{6} \times \frac{7}{8}$$

$$\frac{4}{9} \times \frac{3}{5}$$

Expected student response:

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$$

$$\frac{5}{6} \times \frac{7}{8} = \frac{5 \times 7}{6 \times 8} = \frac{35}{48}$$

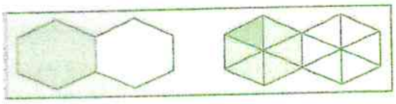
$$\frac{4}{9} \times \frac{3}{5} = \frac{4 \times 3}{9 \times 5} = \frac{12}{45}$$

Higher-Level Demands

Procedures With Connections

Using pattern blocks, if two hexagons are considered to be one whole, find $\frac{1}{6}$ of $\frac{1}{2}$. Draw your answer and explain your solution.

Expected student response:



First you take half of the whole, which would be one hexagon. Then you take one-sixth of that half. So I divided the hexagon into six pieces, which would be six triangles. I only needed one-sixth, so that would be one triangle. Then I needed to figure out what part of the two hexagons one triangle was, and it was 1 out of 12. So $\frac{1}{6}$ of $\frac{1}{2}$ is $\frac{1}{12}$.

Doing Mathematics

Create a real-world situation for the following problem:

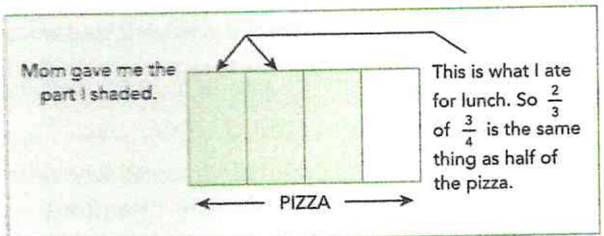
$$\frac{2}{3} \times \frac{3}{4}$$

Solve the problem you have created without using the rule, and explain your solution.

One possible student response:

For lunch Mom gave me three-fourths of the pizza that we ordered. I could only finish two-thirds of what she gave me. How much of the whole pizza did I eat?

I drew a rectangle to show the whole pizza. Then I cut it into fourths and shaded three of them to show the part Mom gave me. Since I only ate two-thirds of what she gave me, that would be only two of the shaded sections.



Source: Smith and Stein (1998). Used with permission.

Figure 3.3