THE MAIN IDEA

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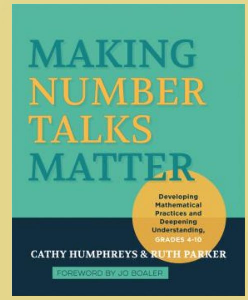


File: Teaching Mathematics

Making Number Talks Matter:

Developing Mathematical Practices and Deepening Understanding, Grades 4 - 10

By Cathy Humphreys and Ruth Parker (Stenhouse Publishers, 2015)

S.O.S. (A Summary of the Summary)

The main ideas of the book:

~ Without much planning, Number Talks are a great, short routine that help students develop number sense through mental math and explaining their reasoning.
~ Because of the real focus on math concepts (not math procedures), this routine has helped dramatically change teaching and learning in math classrooms.

Why I chose this book:
I'm a huge fan of mental math to develop number sense. Number Talks is an easily implementable 15-minute routine to get math teachers to regularly incorporate number sense into their teaching. It can be used alongside any math program.

As math educator, Jo Boaler, states: "Number talks are the best pedagogical method I know for developing number sense and helping students see the flexible and conceptual nature of math."

Although one of the authors (Ruth Parker) created Number Talks in the early 1990s with a colleague, it is still not a widespread practice. In fact, I am amazed at the number of math classes I observe in which teachers teach students to follow a procedure rather than to think and understand the concepts.

It's just much easier to walk students through a series of steps in a math problem than to elicit the type of math thinking that comes from conceptual understanding. This book provides a structure to help teachers ensure they include conceptual math thinking and discussing *every day*.

Both Humphreys and Parker have taught math and led math professional development for decades. Although the book states that it's for teaching math in grades 4 to 10, the ideas work with all grades. The summary is shorter than usual because this month The Main Idea is providing summaries of several books about mathematics instruction.

The Scoop (In this summary you will learn…)

✓ The benefits of doing daily Number Talks

✓ The guidelines to successfully implement Number Talks

✓ How teachers at the middle and secondary levels can make use of Number Talks

✓ Examples of what Number Talks look like and sound like

✓ Professional development suggestions from The Main Idea for implementing the ideas in the book

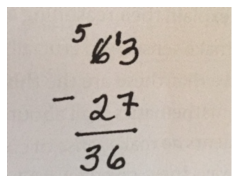
Introduction and Chapter 1: What are Number Talks? Why are They So Important?

By including a short, fifteen-minute routine called Number Talks, teachers can help students learn to reason mentally with numbers – a foundational skill often missed in school. In fact, it is not uncommon to hear middle and high-school teachers lament the fact that their students never developed their mathematical understanding. Teachers have found that without much preparation, their use of Number Talks -- mentally solving and discussing math problems – has led to dramatic changes in both teaching and learning. Another benefit is that Number Talks have altered the way students view mathematics and their ability to succeed with it. In fact, some students enjoy this part of class so much, if the teacher forgets to do conduct a Number Talk, they remind them!

Rationale
Number Talks are needed because traditionally, students learn the steps to a problem without ever understanding the reasoning behind those steps. Take a look at this example of a common mistake:

$$\frac{1}{3}+\frac{1}{3}=\frac{2}{6}=\frac{1}{3}$$

This middle school student *almost* gets the steps right, but he adds the denominator. What's truly upsetting is that the answer does not make any sense at all – how can 1/3 plus 1/3 give you 1/3?! And yet the answer raised no red flag for the student because there was no attempt to *understand* the problem, just complete the steps. This does not mean that the algorithm (the procedure or the steps) is not important. The concern is when procedures are carried out in isolation, without any accompanying understanding. The following problem is one in which the student relies completely on the algorithm. The student is only able to explain the steps she took, "You can't take 7 from 3, so borrow 1 from the 6, and make it a 5. Put the 1 by the 3…"



There are several problems with an overreliance on the procedure. In this case, the student's "reasoning" is not entirely accurate. First, you *can* take 7 from 3 (the answer is negative 4). In addition, you aren't really taking "1 from 6" you are converting one of the tens in sixty. Instead, and the Standards for Mathematical Practice confirms this, we want students to attend to "the *meaning* of quantities, not just how to compute them." With Number Talks, students are given a problem – like the subtraction problem above – but they must calculate the answer in their heads (so they can't use the standard algorithm) and explain their reasoning. Below are some of the ways students solved 63 – 27 and the reasoning they gave during Number Talks:

I took 30 away from 63 and I got 33. Then I added 3 back on because I took too many away. And 33 + 3 = 36. 63 – 27 63 – 30 = 33, plus 3 more = 36	*I added 3 to 27 to get 30, and then I added 3 to 63 and got 66. And 66 – 30 is 36.* 63 – 27 27 + 3 = 30 and 63 + 3 = 66 66 - 30 = 36

When students are given a mental math problem like the one above, and asked to explain their thinking, they benefit in many ways. The authors have found that with Number Talks, students feel they can be competent math thinkers even if they are not fast, are more willing to persevere, and have more confidence in themselves to reason mathematically.

Chapter 2 & Chapter 3: Getting Started and Guidelines for Successful Number Talks

Getting Started
Note that this book doesn't provide a definitive step-by-step guide for implementing Number Talks, nor does it include a set of problems to use. Instead, there is a rough outline of what the routine looks like, a set of guidelines to follow, and suggestions for how and why to choose problems. Below is a general outline of what the routine looks like:

• Students put away paper and pencils and indicate they are ready to conduct mental math.
• The teacher puts a problem on the board or screen – preferably written *horizontally* to discourage a focus on procedures.
• The teacher observes while students take the time they need to solve the problem. Students put up thumbs when they are done.
• The teacher takes *volunteers* who are willing to share and the teacher records only their answers on the board.
• Once all answers are up, the teacher asks who is willing to explain *why* their process makes sense (*not* the steps of a procedure).
• As students describe their strategies, the teacher records their thinking (in a format like the explanations of 63 – 27 above).
• This is the trickiest step: the teacher works with each student's thinking. While there is no set approach, the goal is to have the students communicate clearly. The teacher might ask, "Does anyone have a question for…?" "Can you say more about…?" "Can someone else explain _____'s strategy in your own words?" "What connections do you notice among the strategies we've discussed?"

Before starting, keep in mind that this is quite a transformation for some students. They are used to being given the steps to follow to solve a problem, and then following those steps. Now *they* are the ones who are supposed to think and explain. It is no longer enough to know *how* to do a math problem; now they must understand and be able to explain *why*. Furthermore, our own roles as teachers are reversed from what we've known. Rather than explaining ideas clearly, we must facilitate students coming up with their own thinking. Once we start to explain, we rob them of their own thinking. We must break the habit of doing the thinking for our students, not something that comes naturally to many of us. To be fully prepared for this new role, first we must choose problems that build on students' already existing mathematical understandings. The book has a sample first math problem you can use to introduce Number Talks. Second, we need to prepare so we don't jump in and explain. Below are some questions to guide planning; you can use the template to jot down notes before implementing a Number Talk routine for a specific day.

Planning a Number Talk
1. Anticipate possible strategies students might use for solving the problem:
2. How will you record the different strategies?
3. What questions might you ask to uncover student thinking?
4. Reflect on what you want to remember, what problem you might do next, and why.

Guidelines for Successful Number Talks
The book includes 23 guidelines explained over two chapters that will help make Number Talks successful. These guidelines are condensed and combined into ten basic areas below:

1. <u>Be comfortable with wait time</u>. Students are used to teachers posing a question and then answering it themselves when students do not immediately offer a response. Wait, wait, and then wait some more for students to do the thinking they need to answer questions. Remember there are two opportunities for wait time: one after you've asked a question, and again after a student has responded.

2. <u>Encourage students to explain concepts (why) not procedures (how)</u>. Students are used to explaining the *steps* they've followed. We need to press them and probe their answers to get at conceptual understanding. Middle and high school students are particularly used to listing the steps of an algorithm they've completed. We need to help them make sense of those steps. Today it is no longer sufficient for students to know *what* they did, now they must know *why* their procedures work.

3. <u>There is no one right approach and mistakes are opportunities to learn</u>. When you start Number Talks, students will likely come up with only one way to solve a problem. However, as the teacher emphasizes that there are different ways to understand and solve problems, students will gradually come to see that there is no "right" way. For this reason, it is important for the teacher to record different strategies on the board to demonstrate different ways of approaching the problem. Students must be able to work with numbers flexibly and understand that different people see problems in different ways. Furthermore, when there are multiple answers up on the board (some of which are wrong), the teacher can emphasize that this gives the class a chance to do what mathematicians do: convince skeptics and learn something new. Mistakes help us better understand mathematical concepts.

4. <u>Learn to listen</u>. When students begin to speak, you may think you already know what they are going to say. Try not to jump to conclusions. Use your questioning in an authentic way to try to understand your students' thinking, not narrow it. Listen carefully *to* students rather than *for* what you are hoping they will say. Look at the difference between a question like, "Why did you divide by ten?" which opens up thinking versus, "You took away thirty, so you had to put two back, right?" which is really just an explanation in disguise as a question. Use questions to help students make sense of math *in their own way*. Listen so you can better understand *their thinking*.

5. <u>Do Number Talks regularly and mathematical understandings will develop over time</u>. In order to develop a deep understanding of math, students need to encounter an idea multiple times in a variety of ways. Furthermore, many students are missing basic, foundational knowledge in math. For this reason, it can take quite a while for students to develop number sense. If you commit to doing Number Talks regularly – perhaps every day for the first two weeks – this will give students the time they need to develop their mathematical ideas.

6. <u>Help students express themselves more clearly</u>. First, explain to students why communicating their thinking is important. Encourage them to speak loudly and avoid the use of "it" ("I multiplied *it* by 5.") Finally, make sure students are using academic language rather than, "I timesed 2 and 10." Their use of mathematical language will improve over time.

7. <u>Get students to talk to one another</u>. To get yourself out of the usual spotlight so students can interact with one another, physically move to the side of the room. All students have mathematical ideas worth listening to, and we can't hear them if we are busy intervening with our own thoughts. While we may be anxious to teach students the most "efficient" approach to a problem, if a student does not understand this approach or simply does not think the way we do, then it is not the best approach for this student. One of the most important goals is to help students develop social and mathematical agency. The only way we can help students defend their ideas, change their minds when convinced by reason, and take responsibility for their choices, is if we step out of the way.

8. Know what to do when students are stuck. After establishing the routine of Number Talks, there are times when students are stuck and we need to jump in to share a method that is efficient and mathematically interesting. At other times, our students may be so stuck that we realize the problem is just too difficult. For this reason, it is helpful to always have backup problems to use.

9. Know that confusion and struggle are a natural, necessary, and even desirable part of learning mathematics. As teachers we often rush in to save students from confusion. Instead, we need to encourage students to be willing to struggle and persevere. This may feel uncomfortable because many of us have been taught that our job is to explain procedures clearly. Instead, sometimes confusion and persistence can help students better understand mathematical relationships so we need to encourage students not to give up!

10. Create a learning environment where all students feel safe to share their mathematical ideas. Not every student needs to talk in every session. There are reasons students may not want to share at a particular time. This runs counter to current accountability practices like cold calling. However, psychology researchers have found that stress interferes with mathematics performance, so instead find ways to encourage students to share without putting them on the spot. Students must feel safe during Number Talks if they are going to share.

While there is no step-by-step process, the guiding principles above will help teachers plan Number Talks and make appropriate minute-by-minute decisions as they implement Number Talks.

Chapter 4 – Chapter 8: Number Talks in Action with Sample Math Topics

Each of the next five chapters introduces one mathematical operation or topic (subtraction, addition, multiplication, division, and fractions/decimals/percents), and shows teachers:

- Four or five different number sense strategies to solve problems
- An example of how a student might describe the strategy
- How you might record students' thinking in front of the class
- Sample problems to get you started along with increasingly challenging problems
- Vignettes from actual classroom interactions between teachers and students

While the math topics above might not seem to suit high school students because they don't ostensibly address "high school content," understanding how to reason with numbers lies at the foundation of algebra. Furthermore, any time numbers are involved in a formula, expression, or equation, you can use Number Talks to help your students learn to approach problems with reason, rather than simply jumping into performing the procedural steps. For example, students might approximate the volume of a prism or mentally calculate the measure of a third angle in a triangle rather than jumping into using a formula.

Each chapter delves into the mathematical details of doing Number Talks with one topic. This is fascinating reading for math teachers, but may be too detailed for school leaders, so it is not included here (pass it along to your math team!) To give an example of the contents, below is an outline of the chapter on subtraction.

First, the chapter explains that subtraction may not seem appropriate for older students, but it is key to understanding distance, the slope of a line, the distance formula, and even for finding the area under a curve (calculus!) The chapter then takes the sample problem, 63 – 28, and provides five strategies for solving this one problem. For example, you could round the subtrahend to a multiple of 10, subtract, and adjust (so you would round 28 to 30, subtract that from 63 which gives you 33, then adjust the answer by adding back the two because you subtracted "too many" by subtracting 30, so you get 28). The authors caution that although some of these strategies might be new to you, don't rush to "teach" them because students often come up with the strategies on their own.

After explaining each strategy in depth, there are suggestions for how to choose *good problems* to use with your students and which problems might be best (such as 13 – 9 and 5.14 – 4.6). The chapter also includes two vignettes of teachers interacting with students while discussing the strategies they used to solve a subtraction problem mentally (a fifth grade class discusses strategies for 43 – 28 and a seventh grade class discusses strategies for 3.87 – .79). Below is an excerpt of a student explaining her approach to 43 – 28

> Jennifer: I know my answer is wrong, but I can't figure out why.
> Ms. Young: Do you want to share what you did?
> Jennifer: I took 30 away from 43 and that was 13. Since I added 2 to 28, I took the 2 away from 13 and I got 11.
> Ms. Young: Why did you add the 2?
> Jennifer: I added it to 28 because 30 was easier to take away.
> Ms. Young: So when you took away 30, did you take away too many or too few?
> Jennifer: I took away too many.
> Ms. Young: You took away too many. So will you have to take away more, or will you have to put some back? (Or if Ms. Young had had more time for Number Talks that day, she might have asked the class to try to figure out what Jennifer had done wrong.)
> Jennifer: Well… I have to take away what I added… Oh wait. No. Now I see what I did wrong. When I took away 30, I took off 2 too many, so I have to add them back. Now I see the answer is 15.

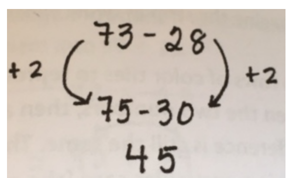
Overall, the goal is to have students learn to reason flexibly and to delve into the *concepts* of mathematics, including those that go beyond subtraction. Once the teacher has learned about students' thinking, he can plan each future Number Talk to build on what students have learned and deepen their understandings about how numbers and operations work.

Chapter 9 – Chapter 11: Investigations, Obstacles, and a Conclusion

Number Talks Can Spark Investigations
When mathematicians notice something interesting in math, they wonder, "Why is this happening? How does this work?" and "Will it always work?" So, what do they do? They *investigate*. When these questions arise during Number Talks, you have the opportunity to start an investigation for students to develop a deeper understanding of the relationship between mathematical ideas and numbers. At first this will be odd for students who have been taught to listen to the teacher and replicate the steps in a math problem rather than truly asking "why" and investigating. These investigations take longer than the typical time allotted for Number Sense (sometimes more than a class period rather than fifteen minutes), and involve the following steps: 1) conducting the investigation yourself to anticipate issues, 2) posing the investigation to share the strategy the class will focus on, 3) small-group work to try to understand the strategy with manipulatives, and 4) whole-group processing and wrap-up to build mathematical knowledge.

The book includes a number of sample investigations into math strategies, and below is an example of one called: *Will it Always Work? Investigation 1: "Same Difference" in Subtraction.* The example problem is 63 – 29 and the strategy to be investigated is called "Same Difference." This is represented by the thinking, "I made the 29 into 30 and the 63 into 64, so I changed the problem to 64 – 30; my answer was 34." Bring graph paper, color tiles or paper squares, rulers, and scissors for students to demonstrate the strategy.

Pose the Investigation: Do a Number Talk for a similar problem, 73 – 28. If no one presents the Same Difference strategy above and changes it to 75 – 30, suggest that some people solve it this way and record it like this:



Small-Group Work: Distribute the materials listed above and let small groups work on the problem. Allow them to work without jumping in, but feel free to ask some questions, such as: "Can you also subtract the same quantity from both numbers?" "Does this strategy work for addition?" and "Will it work with decimals? Fractions?"

Whole-Group Processing: Have students share their findings on a document camera. For this problem, you might see:
• Students make a number line and show how a cut-out piece of paper can slide up and down the number line showing that changing the original numbers doesn't change the distance between them.
• Students may use algebra to show that $a - b = (a + c) - (b + c)$.
Then students discuss whether this will always work and record the big ideas from this problem.

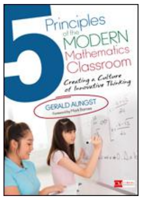
Obstacles
On the one hand, Number Talks are considered a "routine." On the other hand, there is nothing routine about them. Teachers need to think on their feet to decide how to record students' strategies, what to ask, how to respond to students, and more. There is no right answer and good teachers often make different decisions. However, the authors have found that certain issues arise repeatedly, and have addressed those issues in an FAQ section at the end of the book. Read Chapter 10 if you want to learn suggestions for common bumps in the road such as: *What if I don't understand what a student is saying? How can I get my students to move beyond the traditional algorithm? What do I do if I don't know how to record a student's thinking?* and *What should I do when a student's answer or method is wrong?*

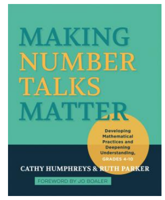
The authors believe that this is a fundamental shift in the way we teach mathematics – away from teaching "what to do" to encouraging students to grapple with the concepts behind the problems. It's not an easy shift to make, so consider finding colleagues who will collaborate with you and support you along the way!

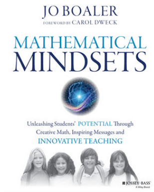
THE MAIN IDEA's PD Suggestions for Math Teachers

Note that these professional development suggestions are for use with these 3 books:

(1) 5 Principles of the Modern Mathematics Classroom, (2) *Making Number Talks Matter,* and (3) *Mathematical Mindsets*







I. LEARN the philosophy underlying THREE books for math educators

A. Teachers Learn the Philosophy in One of the Three Books About Mathematics Education

1. Using a jigsaw approach, divide teachers into three groups and have each group learn about the philosophy in ***one*** of the three books below. While it would be ideal to have teachers read the entire book, given time constraints, teachers can focus on the chapters that contain the underlying philosophy of each book indicated below:

- Gerald Aungst's *5 Principles of the Modern Mathematics Classroom* (chapters 1-2, pp.1-20)
- Cathy Humphreys' and Ruth Parker's *Making Number Talks Matter* (chapters 1-3, pp.1-24)
- Jo Boaler's *Mathematical Mindsets* (chapter 9, pp.171-208)

2. Next, give teachers the chart below with an overview of each author's philosophy. Using the jigsaw approach, have the teachers who read each book introduce the ideas in that book. After all three groups have done this, have the larger group look for what is ***common*** among the three (for example, all believe in the importance of having the right mindset about math).

Humphreys & Parker	Aungst	Boaler
1. Be comfortable with wait time. 2. Encourage students to explain concepts (why) not procedures (how). 3. There is no one right approach and mistakes are opportunities to learn. 4. Learn to listen. 5. Mathematical understandings will develop over time. 6. Help students express themselves more clearly. 7. Get students to talk to one another. 9. Confusion and struggle are natural and necessary to learn. 10. Create a culture where all students feel safe to share their mathematical ideas.	1. Conjecture - Students engage in inquiry, questioning, and problem finding 2. Communication - Students read, write, speak, and listen when reasoning mathematically 3. Collaboration - Students work in pairs and groups 4. Chaos - Class is understandably messy when students truly struggle with mathematical concepts 5. Celebration - The focus is on effort over achievement	1. Everyone can learn math to the highest levels. 2. Mistakes are valuable. 3. Questions are really important. 4. Math is about creativity and making sense. 5. Math is about connections and communicating. 6. Depth is much more important than speed. 7. Math class is about learning, not performing.

3. Conduct a group brainstorm about how a classroom with the mathematical approaches described above would differ from a more traditional math classroom.

Traditional mathematics classroom	Classroom that follows the approaches in the 3 books
Ex. The goal is for students to get the right answers and get them quickly. Ex. Communication is one way with the teacher explaining a procedure or algorithm to the students. Ex. Students mostly work and are assessed individually.	

II. DO some of the math problems that reflect these new approaches

The best (and most fun!) way for math teachers to truly learn the ideas in these books is to have them actually *do* some of the math problems using these new approaches.

A. Participate in a Number Talk as if the Teachers were the Students (Making Number Talks Matter)

1. If you have someone who already has experience with Number Talks or would like to take the lead, read one of the chapters in *Making Number Talks Matter,* and act as the teacher, then you can have that person facilitate this section. If not, simply ask teachers to do the following problem, *in their heads*, without using the traditional algorithm (as a reminder, Number Talks can work with students at all ages as you will see college students discuss the problem below in a video in Step 3!):

18 x 5

2. Next, have teachers share all of the different methods they used to solve this problem (18 x 5) and have the facilitator record (using pictures if possible) all of the different solutions.

3. Have teachers watch and then discuss this video of Jo Boaler introducing Number Talks, Stanford students solving this problem (18x5), and explaining their thinking. (The video is 15 minutes, but you can share just the first 7 minutes): https://www.youcubed.org/from-stanford-onlines-how-to-learn-math-for-teachers-and-parents-number-talks/

B. Experience the Difference Between an "Exercise" and a "Problem" (5 Principles of the Modern Mathematics Classroom)

1. Create sets of cards from 0 to 9 or use decks of cards (the Jack can be the 0). Then divide teachers into pairs and have them play this game. After playing it several times, have them describe the strategy they used to win:

> You and your friends are going to play a game using cards numbered from 0 to 9. On your turn, draw 3 cards from the facedown deck, one at a time. The object is to make the largest 2-digit number with your cards, with one card being discarded. The catch is you must decide where to put each digit before drawing the next: tens place, ones place, or discard. If you draw a 4 as your first card, where should you write it, and why?

2. Next have teachers individually solve the following:

> Miguel collects baseball cards. Last week he had 217 cards. Today, his aunt gave him two dozen more. How many cards does he have now?

3. As a group, discuss how the two math tasks differ. During the discussion, share that Aungst calls the first one a 'problem,' and the second an 'exercise.' Wikipedia defines an exercise in this way:

> An "exercise" is "a routine application of ... mathematics to a stated challenge. Teachers assign mathematical exercises to develop the skills of their students."

4. Ask math teachers to come to future math team (or grade level) meetings with a new problem each time for the teachers to solve. Bring a calendar of all future meetings, sign up teachers, and provide them with these resources to help them find problems:
Bedtime Math, Dan Meyer's Three Act Problems, http://www.math.com/teachers/recreational.html, http://www.insidemathematics.org/problems-of-the-month (problems of the month organized by CCSS), http://math.com/teachers/POW.html, and more.

C. Have Teachers engage in a Rich Mathematical Task (Mathematical Mindsets)

1. Have teachers solve and discuss one of the problems in Chapter 5 of Boaler's book. A few that only involve pencil and paper are:
- How many ways can you create a rectangle with an area of 24?
- Can you make all of the numbers from 1 to 20 using four 4's and any operation. For example:

$$\sqrt{4} + \sqrt{4} + \frac{4}{4} = 5$$

2. Share Boaler's 6 ways teachers can adapt math tasks to boost conceptual understanding and engagement. Discuss what each of these mean and have teachers share ways they have included any of these design elements in math tasks they've assigned.

> 1) Open the math task to include multiple pathways (e.g., "You know the rule for $1 \div \frac{2}{3}$. Now *make sense* of your answer.")
> 2) Make it an inquiry task (e.g., Instead of find the area of a 12x4 rectangle, ask how many rectangles you can find with an area of 24.)
> 3) Ask the problem *before* teaching the method (e.g., Ask calculus students to find the volume of a lemon *before* teaching them how to find the area under a curve.)
> 4) Add a visual component (Have students draw diagrams, pictures, or use objects like multilink cubes and algebra tiles.)
> 5) Make the floor low and ceiling high (Give a problem everyone can solve but extend it by asking those who finish to create a new question that is similar but more difficult.)
> 6) Require students to convince and reason (Require that students give more information than just an answer on its own.)

Ask teachers to look at the rectangle or fours problem they solved above and discuss the design features included in this problem. (Note that not all math problems need to incorporate all 6 of these features to be good problems!)

III. TEACH (or plan to teach) math using a new approach

A. Have teachers create and role-play conducting a Number Talk

1. First, have teachers choose a topic (subtraction, addition, multiplication, division, or fractions/decimals/percents) to plan a Number Talk for. Then have them read the corresponding chapter in *Making Number Talks Matter*.

2. Next, have teachers create a 15-minute Number Talk by choosing a problem (and they can use one from the chapter), anticipating possible strategies students might use for solving the problem, and recording different strategies they think students might use. (Yes, they should actually write these out.)

3. Then have teachers role-play conducting a Number Talk with other math teachers who will act as students. Have one math teacher not participate as a student and instead observe and look for the following in the chart below. Debrief after the role-play, then give everyone a chance to role-play a teacher conducting a Number Talk.

1. Be comfortable with wait time. 2. Encourage students to explain concepts (why) not procedures (how). 3. There is no one right approach and mistakes are opportunities to learn. 4. Learn to listen. 5. Mathematical understandings will develop over time. 6. Help students express themselves more clearly. 7. Get students to talk to one another. 9. Confusion and struggle are a natural, necessary, and even desirable part of learning mathematics. 10. Create a learning environment where all students feel safe to share their mathematical ideas.	Observations of these Guidelines in the Role-Play

B. Have teachers choose a math task they already use with students and increase the level of rigor

1. In *5 Principles of the Modern Mathematics Classroom,* Aungst introduces the difference between an 'exercise' and a 'problem.' If we want to include more "problems" – that is, if we want students to develop a deeper understanding of math concepts -- we need to consider the *rigor* of the problems we assign. One useful tool to use is Norman Webb's Depth of Knowledge. Have teachers look at Webb's descriptions of the different levels of thinking along with the types of math problems that fit each of the first three levels.

Webb's Depth of Knowledge (DOK)	Example from Area and Perimeter	Example from Quadratics
Level 1: Recall and Reproduction (recalling basic facts)	Find the perimeter of a rectangle that measures 4 units by 8 units.	Find the roots of the equation: $y = 3(x-4)^2 - 3$
Level 2: Skills and Concepts (involves some decisions and skills such as comparing, organizing, and estimating)	List the measurements of 3 different rectangles that each has a perimeter of 20 units.	Create 3 equations for quadratics in vertex form which have roots 3 and 5, but have different max or min values.
Level 3: Strategic Thinking (involves planning, evidence and more abstract thinking – such as solving a non-routine problem or explaining the reasoning behind a Level 2 problem)	What is the greatest area you can make with a rectangle that has a perimeter of 24 units?	Create a quadratic equation using the template below with the largest maximum value using whole numbers 1 to 9 no more than once each: $Y = -\square(X - \square)^2 + \square$

2. Next have teachers choose a math task they currently use with students that fits into Level 1 above. Alone or in pairs, have teachers tweak their tasks to create a Level 2 or even a Level 3 problem they could use with their students.

C. Have teachers create norms for the first days back to school AND a rich mathematical task

1. Have teachers read Chapter 9 in *Mathematical Mindsets*. Discuss the chapter, then tweak the 7 norms introduced in this chapter (and outlined on the first page of this PD section (1. Everyone can learn math to the highest levels, etc.)) to fit each teacher's style. Have teachers think through how they want to introduce these during the first days of school. (Note that Boaler has 5 short mindset videos you can use with younger students at youcubed.org.)

2. Have teachers work together to take one of their math tasks and adjust it based on Boaler's 6 design principles in Chapter 5.

IV. ASSESS – What to Look For in Math Classrooms

Have your **leadership team** look at the chart with the overview of the three authors' philosophies at the beginning of this PD outline. Choose *five* areas your math teachers most need to strengthen. Then create a shared list of LOOK-FORS to use when instructional leaders observe math teachers. Brainstorm a list of LOOK-FORS that align with the *five* areas your leadership team chose. Below are a few suggestions:

5 Philosophical approaches we need to strengthen	What to look for when observing math instruction
Ex. There is no one right approach and mistakes are opportunities to learn. (Humphreys and Parker)	• Teachers explicitly tell students their brains grow when they make mistakes. • Teachers spend time with wrong answers rather than jumping to correct ones.
Ex. Chaos - Class is understandably messy when students are truly allowed to struggle with mathematical concepts (Aungst)	• Teachers provide open-ended questions that allow for messiness and failure. • Teachers give students plenty of time to experiment, and keep the stakes low • Teachers don't immediately rescue students who struggle, but allow for struggle.
Ex. Depth is much more important than speed. (Boaler)	• Teachers provide open-ended tasks that emphasize interesting pathways not speed.