PRE-ROUND READING B: MAKING LEARNING VISIBLE THROUGH MATHEMATICAL TALK

BACKGROUND INFORMATION

John Hattie's books are based on three phases of learning: surface learning; deep learning; and transfer Learning These extracts are provided to help make the model clearer.

Almost everything in published research works at least some time with some students. Our challenge as a profession is to be more precise in what we do and when we do it. Timing is everything, and the wrong practice at the wrong time undermines efforts. Knowing when and how to help a student move from sufficient levels of surface learning to deep learning is one of the hallmarks of expert teachers

Surface Learning

It is easy to assume that surface learning means "superficial" or "shallow" or that by surface-level learning we mean rote memorization of procedures and vocabulary that have been traditionally taught at the beginning of the lesson and are disconnected from conceptual understanding. This is not what we mean by surface learning. Rather, the phrase surface learning represents an essential part of learning made up of both conceptual exploration and learning vocabulary and procedural skill that give structure to ideas.

Surface learning talk strategies include:

- Number talks: These short, daily routines provides students with meaningful, ongoing practice with computation. They develop computational fluency because the expectation is that students will use number relationships and structures.
- Guided questioning: designed to help students make sense of what's going on and guide them to draw their own conclusions.
- Teacher modelling: Teachers demonstrate questioning and listening strategies that exemplify excellent mathematical practice as well as give students opportunities to build these habits of mind.

Deep Learning

Deep learning focuses on recognising relationships among ideas. During learning, students engage more accurately and deliberately with information in order to discover and understand the underlying mathematical structure. Students who are involved in deep learning are: displaying, explaining, and justifying mathematical ideas and arguments; communicating; reasoning; and analysing mathematical relationships and connections.

Lessening teacher talk is important at this phase so that students have opportunities to use their prior achievement, understand, sequence and ask questions. Limiting teacher talk is crucial for success in student learning.

Deep learning talk is characterised by accountable talk - a set of discourse expectations for students to enrich maths discussion. *Students should be drenched in accountable talk. It should flood the classroom.* Accountable talk moves are:

- Press for clarification & explanation eg "Can you tell me more"; "Could you describe what you mean?"
- Require justification for proposals eg "How does that support your claim? "Where did you find that information?"
- Recognise & challenge misconceptions eg "I don't agree because ....."; "I think that is a misconception, specifically"
- Require evidence for claims and arguments eg "Can you give me an example?" "How does this evidence support your claim?"
- Interpret & use each other's statements eg "David suggested ....."; I was thinking about Marla's idea and I think..."

Transfer Learning

All the work teachers do is for naught if students fail to transfer their learning appropriately by applying what they have learned in new situations. Hattie explains that transfer learning is about the ways in which students construct knowledge and reality for themselves as a consequence of surface knowledge & deep understanding. Transfer is both a goal of learning and also a mechanism for propelling learning to the next level. Transfer as a goal means that teachers want students to begin to take the reins of their own learning, think metacognitively, and apply what they know to real-world contexts. It also prepares students to move through the progression of mathematical understanding as ideas build on each other across grade levels. Transfer learning is when students reach into their toolbox and decide what tools to employ to solve new and complex problems on their own. When students reach this phase, learning has been accomplished.

Mathematical talk that promotes transfer learning is fueled by discourse that occurs because of rich class discussions. These provide students a chance to shine, to help each other, to hear ideas and make connections. The teacher needs to be an active facilitator who uses questioning to elevate student thinking, presses them for evidence, and requires them to link concepts.

PRE-READING B:

MAKING LEARNING VISIBLE THROUGH MATHEMATICAL TALK

Extract from: *Visible Learning for Mathematics: What Works Best to Optimize Student Learning*

John Hattie, Douglas Fisher and Nancy Frey, Corwin 2017

Making Learning Visible Through Mathematical Talk

We're mindful that these tasks don't exist in a vacuum. These meaningful tasks are fueled by the discourse that occurs in productive class conversations. The language, thinking, and reasoning that occur when discourse happens further contribute to surface, deep, and transfer learning. Discourse is facilitated through purposeful questioning and thoughtful prompts and cues that usually begin with the teacher. Just as there is a need to select tasks that align with learning intentions and success criteria, there are a variety of "math talk" routines and techniques teachers can use to build student understanding and assess how that understanding is developing, and to guide students in self-questioning and self-verbalization to extend metacognition. As teachers consider routines and techniques that facilitate rich classroom discourse, they should also be thinking about the role of discourse in supporting surface, deep, and transfer learning.

EFFECT SIZE FOR SELF-VERBALIZATION AND SELF-QUESTIONING = 0.64

Characteristics of Rich Classroom Discourse

Let's begin by examining the characteristics of classroom discourse that builds student understanding and confidence.

1. Teacher questioning and prompts support students in building understanding based on previous knowledge and making connections rather than the teacher being the authority.
2. Mistakes are valued and seen as opportunities for students to clarify their ideas through discussing and justifying their thinking and listening to the ideas of their peers.
3. Students consider different approaches to the mathematics and how those approaches are similar or different.
4. There is an element of productive struggle among students that is accompanied by perseverance, so that the focus is on how students are going to use mathematics to make sense of the task and how to approach a solution path.
5. Students are encouraged to use a variety of representations to build understanding and justify their thinking.

So, how do we facilitate this kind of rich discourse in our classroom? Read on!

Posing Purposeful Questions

Let's begin by delving into questioning techniques that have different purposes and goals throughout a lesson. In subsequent chapters, we will refer back to these general categories of questions and offer concrete examples of when in the learning cycle these techniques are most appropriate. Purposeful questions serve a variety of outcomes (NCTM, 2014), including the following:

- Encouraging students to explain, elaborate, and clarify thinking to build understanding
- Revealing students' current understanding of a concept
- Making the learning of mathematics more visible and accessible for students

Video 3.2
Questioning That Guides Learning

http://resources.corwin.com/VL-mathematics

By the way, these are not discrete outcomes. It is likely that questions that are intended to support students in building understanding and applying current knowledge to new situations are also providing the teachers information about students' current understanding of a concept. And of course, making the learning of mathematics more visible and accessible for students is the overarching goal for all of our work in recognizing and applying impactful instruction.

Questions That Check, Build, and Deepen Student Understanding

Have you ever started a lesson that builds on previous understandings only to find the students seem to be in the Twilight Zone? Mrs. Norton recalls a situation just like that. After teaching fifth-grade mathematics for many years, she was "promoted" with her students to teach sixth-grade mathematics the following year. When they were ready to extend their understanding of fractional numbers, she thought that there wouldn't be much need for review. After all, fifth grade was the "year of the fraction," and she knew her students had developed understanding through the use of concrete explorations and a variety of applications. So she began the first lesson asking students to solve this example and be ready to explain their thinking:

$$\frac{2}{3} + \frac{1}{2}$$

Imagine her surprise when every student had the answer $\frac{3}{5}$. After she calmed down a bit, she asked her students, "How would you convince me your answer is correct?" Interesting discourse about the sum and whether it was reasonable began to take place. Students began to think about the value of each addend in reference to $\frac{1}{2}$ and one whole rather than a procedure that made no sense. They came to the conclusion that the answer had to be greater than one and that $\frac{3}{5}$ was only a little more than $\frac{1}{2}$; therefore it wasn't reasonable. Several things happened in this lesson. First, and foremost, Mrs. Norton realized that even though she had progressed through the "steps" of concrete, pictorial, and abstract representations in teaching how to add fractions, her students had developed no number sense about fractions. She also realized how powerful the question she asked was for the students so they could begin by taking time to think about what these fractions meant and determine why their answer was not sensible. Subsequent review and lessons built on fraction benchmarks helped to develop deeper understanding of this important concept. In this lesson, Mrs. Norton valued students thinking through the questions she posed. Subsequent discourse provided her with information about her students' misconceptions and, at the same time, pushed student thinking forward.

Purposeful questions promote understanding that can be surface, deep, or transfer depending on where students are in the spiral. Rather than telling students what to do, good questions will move student thinking forward, possibly causing some disequilibrium along the way, so students can work to build on what they know and how to make sense of a given example or problem context. These questions are used not only to prompt student thinking but also to help students explain and justify their thinking. Let's revisit the decimal examples from earlier in the chapter. Good questions will help to promote student understanding even when a concept is new. As students discuss their thinking about where the decimal point belongs in the quotient of $0.735 \div 0.7 = 105$, Mr. Beams's questions require them to think more deeply about what is actually happening with the numbers.

Marcus: I think the decimal point belongs before the 1.

Mr. Beams: How many of you agree with Marcus? *(Pause.)* Marcus, can you explain why you think the decimal point belongs there?

Marcus: Well, both of the numbers I am dividing have the decimal point before the first number, so the decimal point should also be before the first number in the answer.

Mr. Beams: What do the rest of you think about Marcus's reasoning? *(Some nods and other hands are waving to get Mr. Beams's attention.)* What are some other ideas that you have?

Lisa: I think the decimal point belongs between the one and the zero because this problem means how many groups of seven tenths can I make from 735 thousandths.

Bill: But I don't get what you just said. How do you find how many tenths are in thousandths? You didn't convince me that 1.05 is the correct answer.

(A long pause takes place and quiet conversations are happening around the room. Mr. Beams lets this go on for a while and then reconvenes the class by asking the following question.)

Mr. Beams: Can anyone answer Bill's question? How can you explain how many tenths are in 0.735?

Martina: If you look at the place value of 0.735, you see that there is a seven in the tenths place. That means I have seven tenths and some more in that decimal number. If I want to know how many groups of 0.7 I can make, I can determine there is one group of 0.7 and I have a little left to make part of another group. So the decimal belongs after the 1.

Teaching Takeaway

Use questions, prompts, and cues to both help children deepen understanding and also better understand student misconceptions or partial understanding.

Mr. Beams: Can anyone repeat what Martina just said?

Mr. Beams: What does 1.05 mean?

Patricio: It means there is one group of seven tenths in 1.05 and part of another group.

A lot is going on here. Notice that Mr. Beams never tells the students what to do. Marcus's response to the task tells Mr. Beams that some students are looking for a procedure that is neither accurate nor based on mathematical understanding. Mr. Beams allows students some time to talk to each other to make sense of the situation. His questioning carefully draws students back in and allows them to make sense of the example and think of it in terms of previous understandings of division.

While questions that check for understanding are a crucial way to guide learning, the best teachers probe further for more specific information. They don't just want to know whether or not a student understands something; they want to see if the child can explain his or her thinking and apply what is understood, or in this case, misunderstood. If a student doesn't understand, good questions enable teachers to probe deeper in order to find the point at which a misconception, overgeneralization, or partial understanding led students astray. In the back of the teacher's mind is the question "What does this child's answer tell me about what he or she knows and doesn't know?" This is followed by "What question should I ask next?" This is what helps the student begin to move from surface to deep learning.

Funneling and Focusing Questions

You might agree that formal evaluation tools like rubrics are great for longer term, mathematically rich tasks and projects, but it's important to have methods of checking for understanding that you can do anytime you like, regardless of the task. Teachers need to know how much students have actually learned, and how successful a lesson is, in real time so that they can make midcourse adjustments and differentiations. The tools teachers rely on most are the questions we ask of our students. But too often, the questions we pose are interrogative rather than invitational. By this we mean that questions that constrain student responses to short replies are not going to yield much information to the teacher. In addition, these narrow questions don't do much to provoke thinking in students, or to help them notice their own learning. Herbel-Eisenmann and Breyfogle (2005) distinguished between two patterns of teacher-student interactions: funneling questions and focusing questions. **Funneling questions** (Wood, 1998) occur when a teacher guides a student down the teacher's path to find the answer. In these situations, the teacher is doing the cognitive work. **Focusing questions** support students doing the cognitive work of learning by helping to push their thinking forward.

Funneling questions guide students down the teacher's path to find the answer.

Focusing questions allow students to do the cognitive work of learning by helping to push their thinking forward.

EFFECT SIZE FOR QUESTIONING = 0.48

In the book *Principles to Actions,* the National Council of Teachers of Mathematics (NCTM) makes clear the difference between funneling questions and focusing questions. Funneling questions limit student thinking by hinting at an answer, and take the thinking away from the students. Focusing questions encourage students to figure things out for themselves. "What are the measures of central tendency we can use with these data? What are mean, median, and mode?" would be funneling questions, while "What can you tell from the data?" would be a focusing question.

Funneling Questions. Consider how little information is revealed in the following exchange, reported by Herbel-Eisenmann and Breyfogle (2005) as an example of a funneling questioning pattern:

Teacher: (0,0) and (4,1) [are two points on the line in graph B]. Great. What's the slope? *(Long pause—no response from students.)*

Teacher: What's the rise? You're going from 0 on the y [axis] up to 1? What's the rise?

Students: 1

Teacher: 1. What's the run? You're going from 0 to 4 on the x [axis].

Students: 4.

Teacher: So the slope is ______?

Students: 0.25 *(in unison with the teacher).*

Teacher: And the y-intercept is?

Students: 0.

Teacher: So $y = \frac{1}{4}x$? Or $y = 0.25x$ would be your equation. (p. 485)

Funneling questions can create the illusion of deep student learning, but really, they only require the student to know how to respond to the teacher's questioning pattern without understanding the mathematics. These types of questions limit student thinking and leave little opportunity for metacognition. This routine could also be interpreted as scaffolding. But it isn't really, since the questions direct students to what to do rather than giving them opportunities to think about and make connections in ways that effective scaffolding provides. Although the teacher is checking for understanding, the information she gets from her students is limited to whether they are correct or incorrect and doesn't consider anything about understanding or transfer of that understanding.

There can be a role for carefully thought out funneling questions as a new topic is introduced, which has greater impact than a teacher just giving procedural steps to follow. We will talk more about this as we consider surface learning strategies in Chapter 4.

Focusing Questions. The second type of questioning pattern the researchers discuss is called a focusing questioning pattern. These questions are designed to advance student learning, not simply assess it. These are the types of questions you want to ask. Here is the beginning of the same sequence, but this time the teacher goes into a *focusing question sequence* instead of a funneling (Herbel-Eisenmann & Breyfogle, 2005):

Teacher: (0,0) and (4,1) [are two points on the line in graph B]. Great. What's the slope? *(Long pause—no response from students.)*

Teacher: What do you think of when I say slope?

Student 1: The angle of the line.

Teacher: What do you mean by the angle of the line?

Student 1: What angle it sits at compared to the *x*- and *y*-axis.

Teacher: *(Pause for students to think.)* What do you think [student 1] means?

Student 2: I see what [student 1] is saying, sort of like when we measured the steps in the cafeteria and the steps that go up to the music room—each set of steps went up at a different angle. (p. 487)

As the conversation progressed, the students engaged in figuring out how to find the slope. Students who do this are much more likely to understand slope and remember what they figured out a week later, and are much better able to transfer their knowledge—in this case, how to find the slope—to new situations, like projecting sales for a company, constructing a skateboard ramp, or learning how to find derivatives in calculus class.

You've heard the adage that "great teachers don't tell you what to see, but they show you where to look." Focusing questions open up kids' thinking and show them where to look, while funneling questions narrow their thinking in a direction that the teacher has already decided; they tell them what to see. Funneling questions don't allow for multiple paths to solving a problem, for new approaches, or for students to think about their own thinking. With focusing questions, children get to figure it out, so they learn more. They remember the content better, and they can transfer and apply it to new situations. Figure 3.4 contains examples of how funneling questions in mathematics can be transformed into focusing questions.

FUNNELING AND FOCUSING QUESTIONS IN MATHEMATICS

Funneling Questions	Focusing Questions
How do you find the mean of the data? What about the median and the mode? What about the interquartile range?	What do you notice about the data? How would you describe them to someone? What makes you say that? What other ways might you be able to describe them?
How can I get rid of the 2? What do I have to do to the other side? What about the 4?	What do you think about when you see this equation? How do you want to solve it?
How do I find the area of this trapezoid? Do you see the rectangle and the triangles? I can just add them up. How can I find the area of the rectangle?	I want to know the area of this trapezoid, but I'm not sure how to find it. Any ideas? Where should we start?
Let's add these fractions by finding the least common denominator. What's the first step in finding the least common denominator?	What should we do with these fractions? [*Student: "Add them."*] Why add them? [*Student refers to word problem.*] Okay, so how would you add them?

Figure 3.4

Some other useful focusing questions to have in your back pocket are the following:

- What are you trying to find?
- How did you get that?
- Why does that work?
- Is there another way you can represent that idea?
- How is this connected to (other idea, concept, finding, or learning intention)?

Questions that check for understanding are a crucial aspect of visible learning. The best teachers probe deeper for more specific information. They don't just want to know whether or not a student understands something. If the student does, they want to see if the child can explain his or her thinking and apply what is understood. If the student doesn't understand, these teachers probe deeper to find the point at which a misconception, overgeneralization, or partial understanding led them astray. In the back of the teacher's mind is this question: "What does this child's answer tell me about what he or she knows and doesn't know?" This allows the teacher to determine the type of learning that the student needs next.

Teaching Takeaway

Use questions to better understand student misconceptions or partial understanding.

A key to effective checking for understanding is to avoid false positives. In other words, you don't want to fool yourself into believing that your students know something when they really don't. Novice teachers often ask a question, wait for a volunteer to respond, and then think the class gets it because the volunteer has the correct answer. This pattern doesn't work very well to get all students learning. The teachers who rely primarily on volunteers are almost always disappointed when more accurate data prove that the majority of their students haven't learned as much as their handful of volunteers. This is one advantage of having students work in groups. As you walk around and listen to group conversations, pausing to ask probing questions can provide information about where students are in their understanding rather than where one student is.

Here's one last important hint about asking good questions: The types of questions we are calling for are likely not the questions that we experienced from teachers when we were students. It takes thoughtful planning to prepare the kinds of questions that will best support your students' learning while making them more independent learners. Asking good questions models for students the kinds of questions they can ask themselves when they are stuck. Good questions are seldom spontaneous. As you are putting the practice of posing purposeful questions (NCTM, 2014) into action, give yourself time to stop and think about what question you want to ask that serves student learning and fuels constructive communication.

Prompts and Cues

Questions are the starting place that helps teachers check for understanding. Prompts encourage students to do cognitive or metacognitive work. They can take the form of a statement or a question. When Daniel Castillo said to a student who was stuck, "Based on what you know about functions, can that be true?" he wasn't just checking for understanding. He was asking the student to return to her background knowledge. Prompts should challenge students rather than do the thinking for them. **Prompts** are often used to activate background knowledge and interrupt the temporary forgetting of prior knowledge in the face of new learning. Saying "Think about what you already know about

Prompts are questions or statements used to remind students to leverage what they already know in order to think further.

finding a common denominator as you read that question again" can remind them to use what they do know. Prompts are a bit narrower than questions, as they come after you've had a chance to engage with the child using those focusing questions. When questions don't spur action, prompts can move students forward.

Another prompt is revoicing what the student has said to give all students a chance to think about it, clarify whether you have understood the explanation accurately, and give the student talking an opportunity to think about his or her thinking. For example, "So you're saying that we'll have three-eighths left over?" This can be especially powerful if a student's thinking seems unclear, or if he or she spoke in a way that makes it tough for other students to hear (Chapin, O'Connor, & Anderson, 2009). Another move is to ask another student, "Can you say what [student name] just said, in your own words?" This is especially helpful for English learners, and it helps the rest of the group to process what the first student said. Figure 3.5 includes sample prompts with examples.

Cues are more overt attempts to draw attention to relevant information or a certain action needed to move forward.

Cues are more direct and overt than prompts, as they shift the student's attention to the relevant information or study action needed to move forward. Examples of effective cues are when a teacher points to a vocabulary word posted on the wall, to the lesson's learning intentions, to another student who is using her notes, to a figure in the textbook, or to sentence starters on a table tent. If a student is looking at a page that's different from what you assigned, then a verbal cue might be in order, such as a whispered "The class is on the other side of this paper," or even better, "Look around at everyone else's paper." This doesn't take away children's thinking if they've already shown that they're proficient in turning to the right page, since it was probably an error of whoever passed out the papers. Just like questioning patterns, you could imagine funneling cues—"Look at the left side of data table #3 when you're deciding which numbers to use"—and focusing cues—"Think about how you could know which numbers to use" or "Remember that you have resources here to help you." Figure 3.6 includes definitions of several types of cues.

Teaching Takeaway

Use prompts and cues to help students zero in on new learning, remember critical points, and connect to previous learning.

In using prompts and cues, teachers must be careful that they ask all students to think about why their work is correct or incorrect. Teachers can inadvertently create a situation where students know their answer is incorrect because the teacher uses certain prompts or cues that he or she does not use when seeing a correct answer. The prompt "Does that

TYPES OF PROMPTS FOR MATHEMATICS

Type of Prompt	Definition	Example
Background knowledge	Reference to content that the student already knows, has been taught, or has experienced but has temporarily forgotten or is not applying correctly.	• When trying to solve a right-triangle problem, the teacher says, "What do you recall about the degrees inside a triangle?" • As part of their study of solid figures, the teacher says, "Think about what you remember about vertices, edges, and faces."
Process or procedure	Reference to established or generally agreed-upon representation, rules, or guidelines that the student is not following due to error or misconception.	• When a student incorrectly orders fractions thinking the greater the denominator, the greater the fraction, the teacher might say, "Draw a picture of each fraction. What do you notice about the size of the fraction and the number in the denominator?" • When a student is unsure about how to start solving a problem, the teacher says, "Think about which of the problem-solving strategies we have used might help you to get started."
Reflective	Promotion of metacognition—getting the student to think about his or her thinking—so that the student can use the resulting insight to determine next steps or the solution to a problem.	• The student has just produced a solution incorrectly, and the teacher says, "Does that make sense? Think about the numbers you are working with and the meaning of the operation." • A teacher says, "I see you're thinking strategically. What would be the next logical step?"
Heuristic	Engagement in an informal, self-directed, problem-solving procedure; the approach the student comes up with does not have to be like anyone else's approach, but it does need to work.	• When the student does not get the correct answer to a math problem, the teacher says, "Maybe drawing a visual representation would help you see the problem." • A teacher says, "Do you think you might find it easier to begin with a simpler but similar problem? What might that problem look like?"

Source: Adapted from Fisher and Frey (2014).

Figure 3.5

TYPES OF CUES FOR MATHEMATICS

Type of Cue	Definition	Example
Visual	A range of graphic hints to guide students' thinking or understanding	• Highlighting areas within text where students have made errors • Creating a graphic organizer to arrange content visually • Asking students to take a second look at a graphic or visual from a textbook
Verbal	Variations in speech to draw attention to something specific or verbal "attention getters" that focus students' thinking	• "This is important . . ." • "This is the tricky part. Be careful and be sure to . . ." • Repeating a student's statement using a questioning intonation • Changing voice volume or speed for emphasis
Gestural	Body movements or motions to draw attention to something that has been missed	• Making a predetermined hand motion such as equal or increasing • Placing thumbs around a key idea in a problem that the student is missing
Environmental	Use of the classroom surroundings or physical objects in the environment to influence students' understanding	• Using algebra tiles or other manipulatives • Moving an object or person so that the orientation changes or the perspective is altered

Source: Adapted from Fisher and Frey (2014).

Figure 3.6

answer make sense? Really think about it." can be used even following a correct answer to help that learner think about how to justify his or her thinking.

Too often, we ask questions or give students prompts or cues only when they are incorrect. Try asking students a question or providing a prompt when they are correct. Notice if they automatically assume they are incorrect because you stopped to ask a question. Productive questions, prompts, or cues should be a regular part of our instruction moves repertoire!

Video 3.3
Student Discourse That Builds Understanding

http://resources.corwin.com/VL-mathematics