Reasoning in the Australian Curriculum: UNDERSTANDING ITS MEANING AND USING THE RELEVANT LANGUAGE

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The Australian Curriculum: Mathematics encourages teachers to consider seriously the four proficiencies: Understanding, Fluency, Problem Solving and Reasoning. In responding to the reasoning proficiency, many teachers may find that the language of the classroom may well change. In this article, we discuss the meaning given to the term reasoning within the AC:M and elsewhere, share an example of a classroom task with the potential to develop mathematical reasoning, consider the language involved in describing reasoning, and present some survey responses on teachers' current understanding and use of the relevant terms.

The role of reasoning in the teaching and learning of mathematics

In considering the ways in which other writers describe the meaning of reasoning and mathematical actions associated with it, a common language starts to emerge. According to Russell (1999), "mathematical reasoning is essentially about the development, justification, and use of mathematical generalizations" (p. 1). Lannin, Ellis and Elliot (2011) described mathematical reasoning as "an evolving process of conjecturing, generalising, investigating why, and developing and evaluating arguments" (p. 13). Further, they defined generalisation as involving identifying commonalities across cases or extending the reasoning beyond the range in which it originated, … [including] using and classifying the meaning of terms, symbols, and representations" (p. 27). They also distinguish a mathematical justification (which they describe as a logical argument based on already understood ideas, from an argument based on authority, perception, popular consensus or examples).

In the *Australian Curriculum: Mathematics* (AC:M) there is a 'definition' of reasoning. The verbs associated with reasoning include evaluating, explaining, inferring, justifying and generalising. It is stated that students are reasoning mathematically when they:

- explain their thinking;

- deduce and justify strategies used and conclusions reached;
- adapt the known to the unknown;
- transfer learning from one context to another;
- prove that something is true or false; and
- compare and contrast related ideas and explain their choices.

In urging teachers to write and adapt tasks that promote conjecturing and generalising, Kilpatrick, Swafford and Findell (2011) offered the following advice to teachers:

- extend tasks that deal with specific instances to encourage general reasoning;
- provide tasks that promote the development of conjectures. Often these tasks are phrased in general language, such as, "What happens if…?"
- encourage students to justify conjectures and generalisations by using words, numbers, diagrams and symbols to examine the mathematical characteristics and structures in tasks;
- ask students to evaluate whether a student's statement or justification is valid;
- ask students to explain why their statement is true to a student in an earlier grade (e.g., "How would you explain that this is always true to a student in first grade?");
- ask students to answer the questions, "Do you think this will always be true?" "When do you think this will be true?" and "Are there times when this won't be true?";
- develop a classroom culture in which the mathematical correctness of a response relies on the validity of the mathematical justification that is provided rather than some external authority;
- encourage students in small and whole-group settings to share why they think a statement is true and hold all students accountable for understanding;
- establish norms for conjecturing, generalising and justifying that create an environment where students feel safe to share correct and incorrect ideas; and
- encourage students to take risks by sharing their reasoning.

Fraivillig (2004) provided a list with much in common with that of Kilpatrick et al. (2011). The challenge is to find classroom tasks that offer students the opportunities to engage in such actions.

A task with the potential to promote mathematical reasoning

In recent months, we have been using the following task with students in Years 5 and 6. We first became aware of it in Way (2008), and have enjoyed and adapted it in a variety of ways.

After some initial discussion of how students might work out a series of calculations such as 3 + 5 – 6 + 2 and 2 – 5 + 4 – 7 (carefully chosen to bring up the issue of calculations which move students into negative numbers), we posed the following task:

> Write 4 consecutive numbers, in order from smallest to largest. Insert an addition or subtraction sign between each pair of numbers. Calculate the answer, and then do it again for another mix of operations. Write one equation per slip of paper. What patterns do you notice?

This is a richer task than it might initially appear to the reader. Students start to consider different ways they can separate four consecutive numbers with addition and subtraction signs. Eventually, most realise that eight ways are possible. They are then encouraged to organise their eight equations in an order that makes sense to them, line them up beside their partner's set of eight, and stick them down on an A3 page, as in Figure 1.

The students are then invited to consider the two sets of equations and see what they notice. Some students will comment that all numbers on the right hand side of each equation are multiples of 2, some will notice that there seems to be a 0 in each set, as well as a –2 and a –4 in each set.

Others will start to articulate why that

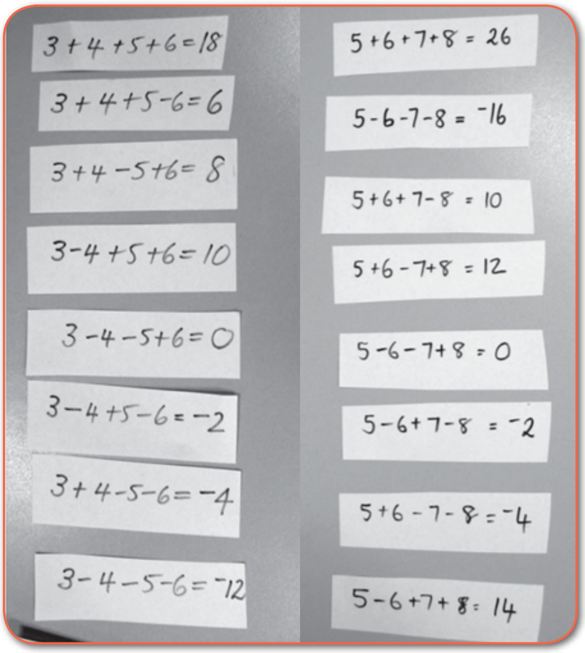

Figure 1

might be. For example, a number of students have commented that any equation with "–, –, +" in that order, will result in zero, but they had difficulty explaining why that might be the case. Indeed, part of the advantage of this task is that students can come to see that such explanations are possible.

We have noticed there is a progression which students may or may not move through:

- Noticing some surface features of the equations ("I have some negative answers and some positives"; "His answers are bigger than mine").
- Noticing more important relationships but not being able to articulate them well ("We both got zeros").
- Starting to conjecture, but lacking the tools or language to explain ("Any set that has a – – + will result in zero")
- Conjecturing and justifying why this conjecture is reasonable ("If four consecutive numbers, written in ascending order, are separated by –, –, and +, respectively, then the result will be zero because the sum of the two outside numbers equals the sum of the two inside numbers").

One issue that needed to be addressed was some confusion between the consecutive numbers themselves and the operations on them. It seemed important to distinguish

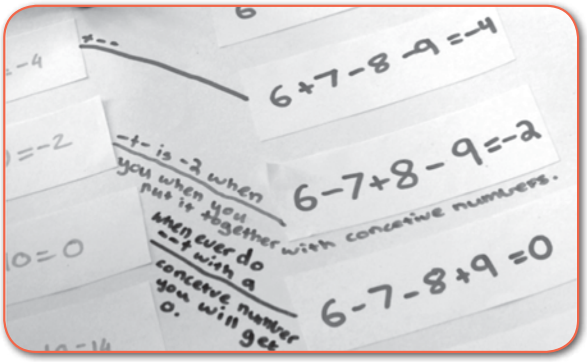

Figure 2

these by verbalising "negative 5 minus (or subtract) negative 4, for example. This may be assisted by placing the consecutive numbers on four separate cards, and the students choosing from different cards which have operation signs on them.

Our experience is that many students appeared to have little experience in the opportunity to conjecture, justify and generalise, or certainly to articulate these processes verbally or in writing. Some examples in Figure 2 illustrate attempts by students to do so.

In relation to the Content Descriptions in the AC:M, the following seem the closest match:

- Year 3: Recall addition facts for single digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation (ACMNA055)
- Year 4: Investigate and use the properties of odd and even numbers (ACMNA071)
- Year 7: Compare, order, add and subtract integers (ACMNA280)

In trying out this task, we realised that while it had the potential to promote reasoning, conjecturing, justifying and informal proof, the students' relative inexperience with this kind of mathematics task made reasoning about their findings challenging. We believe that the task could relate to to all nine elements of Kilpatrick et al.'s (2000) advice to teachers, particularly the opportunity to suggest conjectures and test them out, and the opportunity to generalise.

If teachers are going to embrace the Reasoning Proficiency in the *Australian Curriculum*, then concerted efforts in professional learning settings may be needed to assist teachers in supporting students to develop the type of reasoning advocated by the AC:M. An important element of this is teachers feeling comfortable with the many terms associated with mathematical reasoning.

In the following section, we share data from survey items where teachers were asked to express their own understanding of particular mathematical terms associated with reasoning AC:M. We also discuss the extent to which teachers actually claim to use these terms with their students.

Results

In the Peopling Educational Policy (PEP)[1] Project, we are researching the kinds of support which teachers might require to implement the Australian Curriculum in Mathematics and English. We now discuss findings from the PEP where survey data revealed much about the planning processes of primary mathematics teachers, and, in particular, their sense of the most important ideas underpinning the topics that they teach.

In the survey that was described in earlier articles in this issue, teachers were presented with the definition of reasoning from the AC:M as presented above, and asked the following:

> Rate the extent to which most teachers would find this statement understandable.
>
> On a scale from 1 to 10, with 1 being "difficult to understand" and 10 meaning "easy to understand", the overall mean of the responses was 5.7. More interestingly, the scores were more or less evenly spread across the ten possible categories, indicating a wide range of views on the extent to which the statement is understandable. Some opportunity to explore the meaning of reasoning generally and this definition in particular would be useful for teachers.

The teachers were also invited to indicate which of a range of terms they "regularly use when teaching mathematics". Table 1 presents the frequency at which the 104 teachers chose the particular words.

Table 1. Frequency of teachers indicating regular use of particular reasoning terms (n = 104).

Reasoning adverb	Number of responses
explaining	100
justifying	76
proving	72
reasoning	64
evaluating	48
analysing	47
generalising	35
inferring	30
deducing	24
adapting	24
transferring	20
contrasting	14

Nearly all teachers reported regularly requiring their students to explain something, and around three quarters claimed to regularly ask their students to prove something. The least commonly chosen options were contrasting, transferring and deducing. On one hand, this suggests that some more work on what constitutes explaining and proving might be useful. On the other hand, the potential and examples of contrasting, transferring and deducing may productively be included in teacher education sessions.

The teachers were also invited to indicate:

> How often do you teach students specifically how to use the key terms associated with the language of reasoning? (such as "justify", "conjecture", "investigate", etc.)

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The percentage of the responses in the various categories of frequency are presented in Table 2.

Table 2. Teacher reported frequency at which they teach the language of reasoning (n = 104).

Frequency of use	Percentage of teachers
Once or twice a year	15
Once or twice a term	16
Once or twice a month	20
Once or twice a week	38
Every lesson	12

Again there was the full range of possible responses. Around half of the teachers claimed to incorporate the language of reasoning into their teaching weekly or more frequently.

The teachers were also asked to indicate:

> What percentage of your mathematics lessons have specific lesson goals associated with reasoning as the main focus? [do have]
>
> What percentage of your mathematics lessons should have specific lesson goals associated with reasoning as the main focus? [should have]

Their responses are presented, as percentages, in Table 3.

The diversity of responses is also evident here, and many teachers clearly thought that there should be more reasoning than there was.

Table 3. Percentage of teachers indicating the extent to which lessons "do have" and "should have" reasoning as part of the goals.

% of lessons	Do have	Should have
0	3	0
10	14	3
20	13	5
30	14	7
40	10	8
50	21	15
60	5	8
70	10	10
80	5	12
90	4	10
100	1	21

Conclusion

The Reasoning proficiency in the AC:M is recognised as an important part of mathematics, and one which possibly has been underemphasised in teaching in recent years. The types of actions described in the definitions are important for students at all levels. Our experience is that teachers need support in identifying tasks that prompt reasoning, and also need support in assisting students in becoming familiar with the language that they will need to describe their reasoning. The survey results indicate that teachers incorporate some aspects of reasoning into their teaching, but not others. The results also suggest that teachers recognise the need for more. This will be an important challenge for those who provide and support professional learning for teachers of mathematics.

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